

Physics, Measurements, and Modeling of Wind-Driven Circulation in Upper Klamath Lake, Oregon

**Ralph T. Cheng
U. S. Geological Survey
Menlo Park, California**

Outlines

- I. An overview of Numerical Modeling
Eulerian and Lagrangian Views**
- II. Numerical Methods and the TRIM
family of models**
- III. Using UnTRIM to model Wind-Driven
Circulation in Upper Klamath Lake,
Oregon**

Numerical Modeling of Environmental Flows

**Scales: Physical Properties or
Physical Processes**

Spatial and Temporal

Scales

**Need the Right Model to represent the proper
physical properties and to resolve the physical
processes of the environmental problem**

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

10. The technique was first developed here!

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

- 9. You can run the code without the manual!**
- 10. The technique was first developed here!**

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

- 8. There are no more bugs in the code, only undocumented features!**
- 9. You can run the code without the manual!**
- 10. The technique was first developed here!**

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

- 7. User Friendly, Minimal learning curve!**
- 8. There are no more bugs in the code, only undocumented features!**
- 9. You can run the code without the manual!**
- 10. The technique was first developed here!**

10 Great Lies of CFD Codes — Los Alamos

(Food for thought in David Letterman Style)

- 6. All physics are compatible!**
- 7. User Friendly, Minimal learning curve!**
- 8. There are no more bugs in the code, only undocumented features!**
- 9. You can run the code without the manual!**
- 10. The technique was first developed here!**

10 Great Lies of CFD Codes

– Los Alamos

(Food for thought in David Letterman Style)

5. Robust and accurate!

10 Great Lies of CFD Codes

– Los Alamos

(Food for thought in David Letterman Style)

- 4. Executable on all machines without modifications!**
- 5. Robust and accurate!**

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

- 3. Standardized graphics output, compatible with third party post-processors!**
- 4. Executable on all machines without modifications!**
- 5. Robust and accurate!**

10 Great Lies of CFD Codes

— Los Alamos

(Food for thought in David Letterman Style)

- 2. The manual has everything you need to run the code!**
- 3. Standardized graphics output, compatible with third party post-processors!**
- 4. Executable on all machines without modifications!**
- 5. Robust and accurate!**

10 Great Lies of CFD Codes

– Los Alamos

(Food for thought in David Letterman Style)

- 1. It will solve your problem without modifications!**
- 2. The manual has everything you need to run the code!**
- 3. Standardized graphics output, compatible with third party post-processors!**
- 4. Executable on all machines without modifications!**
- 5. Robust and accurate!**

Lagrangian point of view:

Reference frame is enclosing the mass.

The coordinates are moving with the center of the mass.

Eulerian point of view:

**Reference frame is fixed in space,
the mass travels through the control
volume.**

The coordinates are fixed in space.

SAN FRANCISCO BAY WIND PATTERNS

[SITE MAP](#)



San José State
UNIVERSITY

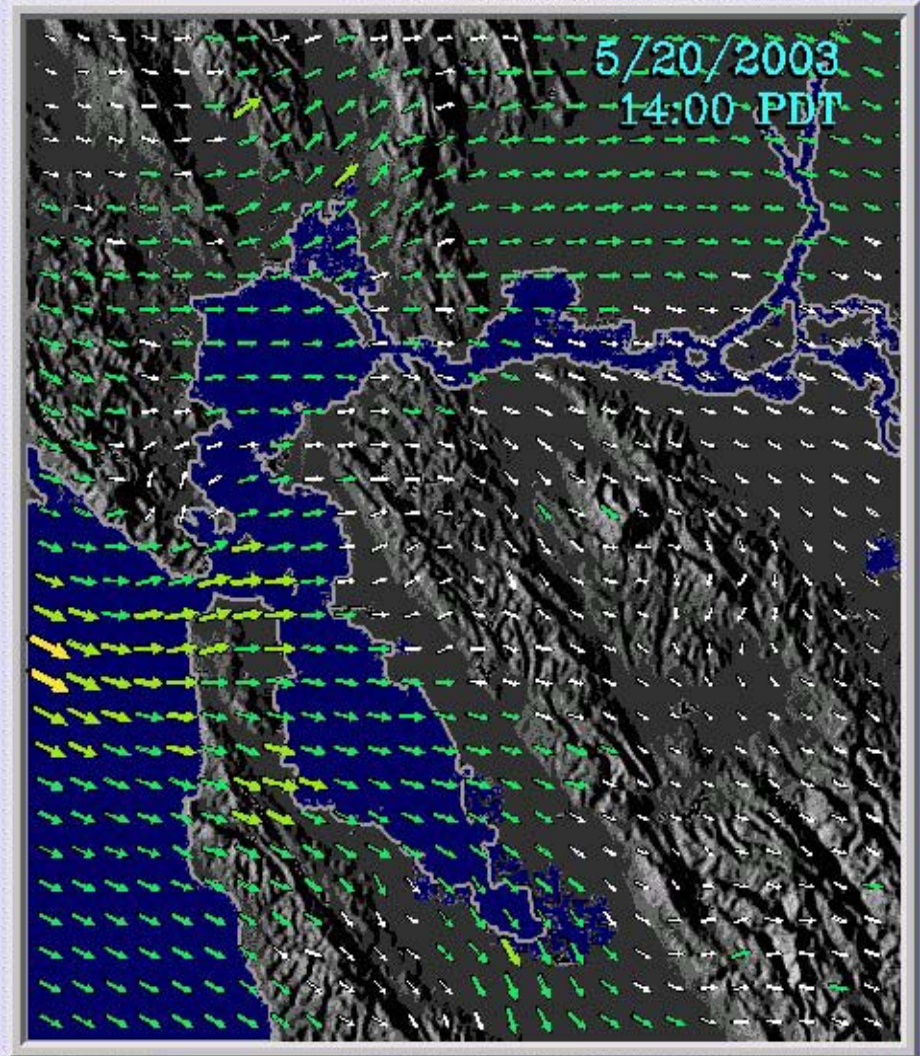


[Francis L. Ludwig](#)

[Observed Wind
Over S.F. Bay](#)

[How are the winds
generated?](#)

[SFPORTS: Tides
& Winds, Currents](#)



Eulerian Representation

Eulerian Variable: $\theta = \theta(x, y, z, t)$

Lagrangian Representation

SAN FRANCISCO BAY WIND PATTERN STREAKLINES

[SITE MAP](#)



San José State
UNIVERSITY



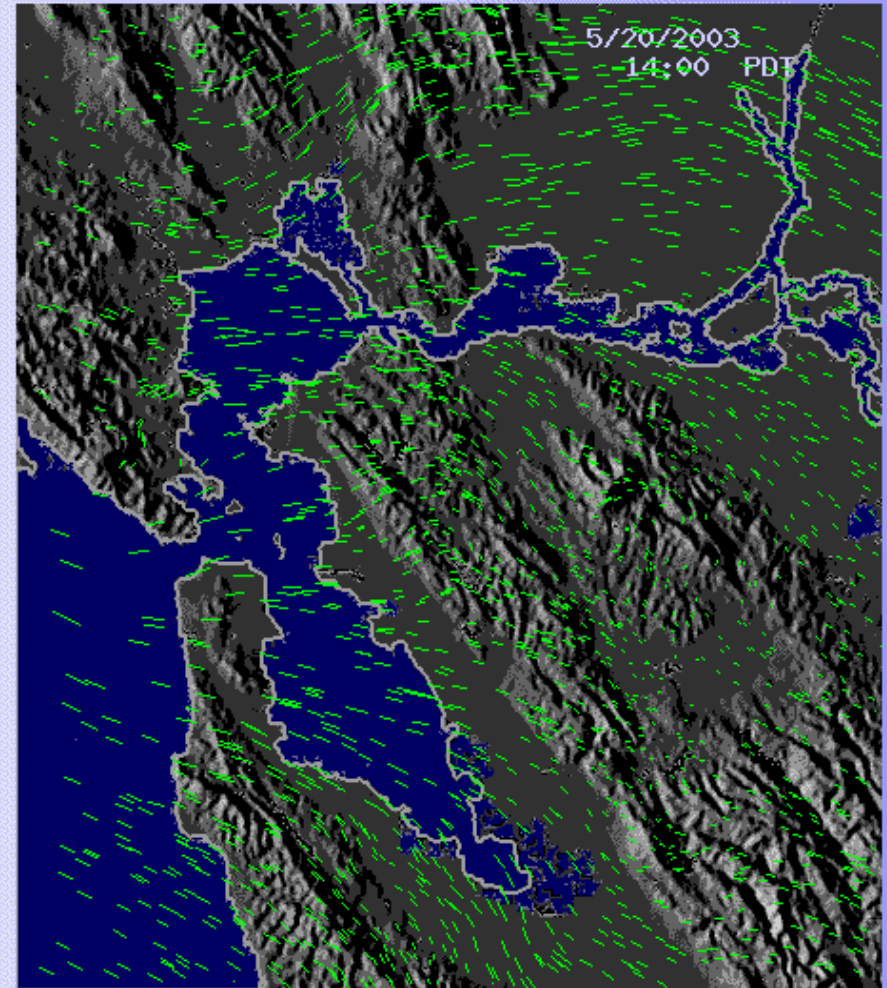
[Francis L. Ludwig](#)

Click to start/stop

Kudos to
[Nick Thompson](#) for
this applet -- See
Description **Below**

[Modeled Wind
Field Over
S.F. Bay](#)

[SFPORTS: Tides
& Winds, Currents](#)



Lagrangian Variable: $\theta = \theta[\vec{X}_o(t_o), \vec{X}(t), t]$

Observations:

Lagrangian Point of View:

Physics is clear

Discrete particle dynamics

Measurement difficulties

Hard to quantify measurements

Eulerian Point of View:

Continuum

Operational Convenience

Easy to organize “information”

Euler-Lagrangian Transformation:

Substantial Derivative

$$\frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z}$$

Some Common Measurement Techniques:

Lagrangian Reference Frame:

Lagrangian Variable: $\theta = \theta[\vec{X}_o(t_o), \vec{X}(t), t]$

Most flow visualization techniques

Dye studies, drifters

Long-term path of water ‘mass’

Measurement Difficulties, Hard to quantify measurements

Eulerian Reference Frame:

Eulerian Variable: $\theta = \theta(x, y, z, t)$

Fixed Current Meter, CTD moorings

Cruising and Profiling ADCP, CTD

HF Radar for surface current and waves

Operational Convenience, Easy to organize “information”

Numerical Methods

Lagrangian Point of View:

Clear Physics

Difficulties to quantify measurements

Eulerian Point of View:

Continuum, Operational Convenience

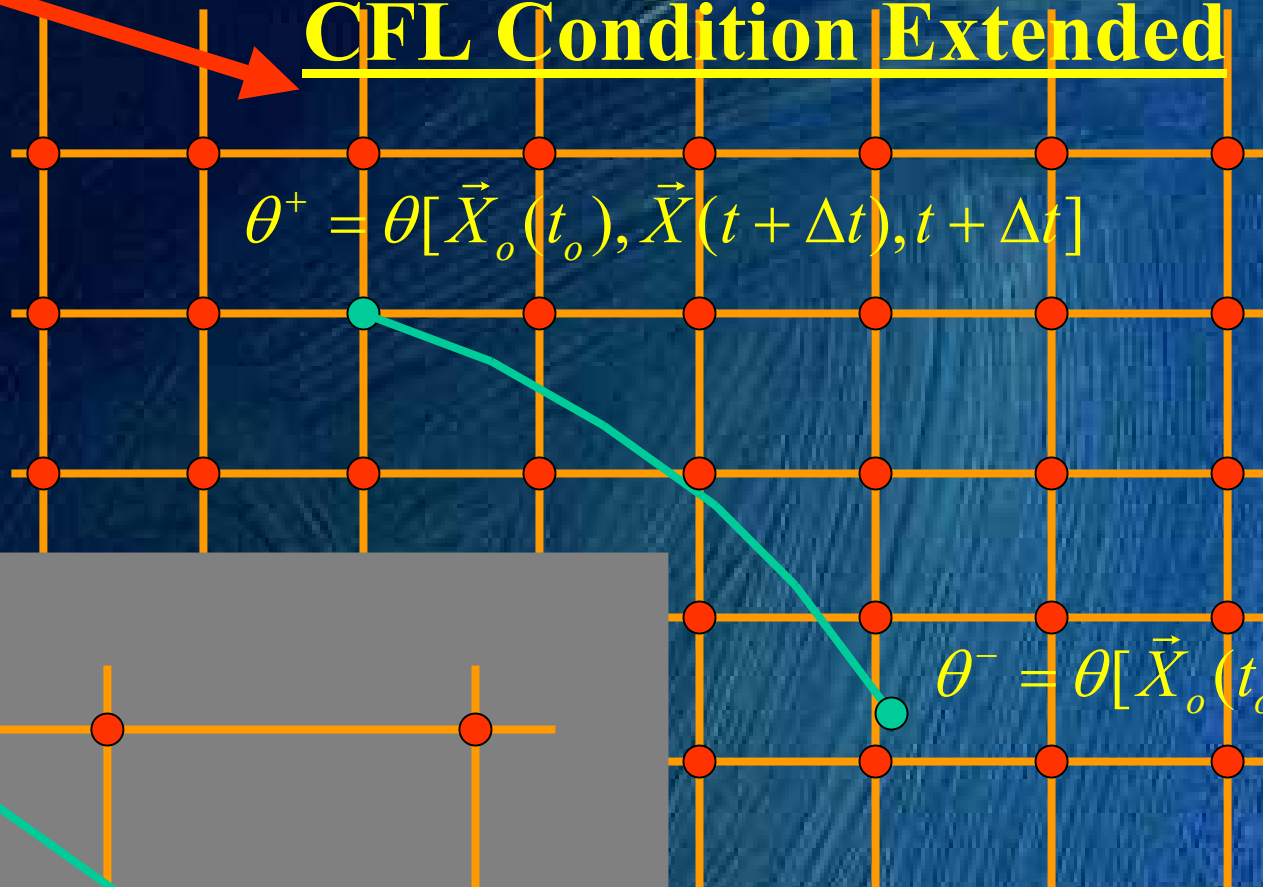
Easy to organize “information”

Substantial Derivative: Euler-Lagrangian Transformation

$$\frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z}$$

$$\frac{D\theta}{Dt} = \frac{\theta^+ - \theta^-}{\Delta t} = \frac{\theta[X(t_o + \Delta t)] - \theta[X(t_o)]}{\Delta t}$$

Eulerian-Lagrangian Approach: CFL Condition Extended



● Eulerian Data

**Origin of Numerical Dispersion:
Interpolation of Eulerian Data to
Lagrangian Point**

Numerical Modeling

25 years Collaborations with

Vincenzo Casulli



Cheng, R.T., and Casulli, V., 1982, On Lagrangian residual currents with application in South San Francisco Bay, CA, Water Resources Research, v. 18, No. 6, p. 1652-1662.

Numerical Modeling

25 years Collaborations with Vincenzo Casulli

The TRIM Family of Models From TRIM to UnTRIM

- **Solution of Shallow Water Equations**
- **Transient, Multi-Dimensional (3D, 2D, 1D)**
- **Simultaneous Solution of Transport Variables**
- **Semi-implicit Finite-Difference Method**
- **Boundary Fitting Unstructured Grid Mesh**

Formulating the Algorithm for a Numerical Model

Desirable Properties of a Numerical Model:

1. Stability
2. Accuracy (Require compromise)
3. Efficiency

Numerical Algorithm



From PDE to Discrete Algebraic System:

Spatial discretization:

Finite difference, Finite Element, Finite Volume

Temporal discretization:

Explicit scheme, Implicit scheme, Semi-implicit

Numerical Foundation of TRIM (Background)

Casulli, V., 1990, **Semi-implicit** Finite-difference Methods for the Two-dimensional Shallow Water Equations, J. Comput. Phys., V. 86, p. 56-74.

Desirable Properties of a Numerical Model:

1. Stability
 2. Accuracy
 3. Efficiency
- (Compromise)

Stability Analysis: Gravity wave terms and velocities in Continuity Eq. control the numerical stability

Method of Solution:

1. Treat those terms implicitly, and the remaining terms explicitly.
2. Substituting momentum Eqs. into continuity Eq., resulting a matrix equation that determines the water surface of the entire domain.

2D Depth-Averaged Shallow Water Equations

Continuity Eq.:
$$\frac{\partial \zeta}{\partial t} + \frac{\partial [(h + \zeta)U]}{\partial x} + \frac{\partial [(h + \zeta)V]}{\partial y} = 0$$

X-Momentum Eq.:

$$\left(\frac{DU}{Dt} \right) - fV = -g \frac{\partial \zeta}{\partial x} + \frac{1}{\rho_o (h + \zeta)} (\tau_x^w - \tau_x^b) + A_h \nabla^2 U - \frac{g}{2\rho_o} (h + \zeta) \frac{\partial \rho}{\partial x}$$

Y-Momentum Eq.:

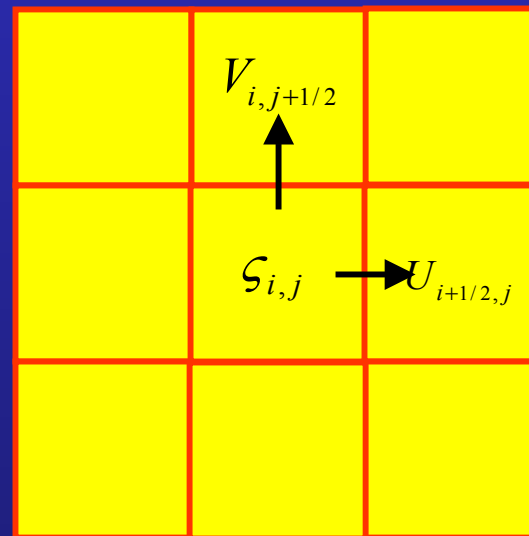
$$\left(\frac{DV}{Dt} \right) + fU = -g \frac{\partial \zeta}{\partial y} + \frac{1}{\rho_o (h + \zeta)} (\tau_y^w - \tau_y^b) + A_h \nabla^2 V - \frac{g}{2\rho_o} (h + \zeta) \frac{\partial \rho}{\partial y}$$

Eulerian-Lagrangian Method (ELM) => Stability (von Neumann)

X-Momentum Eq.:

$$\frac{DU}{Dt} - fV = -g \frac{\partial \zeta}{\partial x} + \frac{1}{\rho_o(h + \zeta)} (\tau_x^w - \tau_x^b) + A_h \nabla^2 U - \frac{g}{2\rho_o} (h + \zeta) \frac{\partial \rho}{\partial x}$$

Semi-implicit FD: Algebraic Eq. of $\zeta_{i,j}^{n+1}, U_{i+1/2,j}^{n+1}, \zeta_{i+1,j}^{n+1}$



Y-Momentum Eq.:

$$\frac{DV}{Dt} + fU = -g \frac{\partial \zeta}{\partial y} + \frac{1}{\rho_o(h + \zeta)} (\tau_y^w - \tau_y^b) + A_h \nabla^2 V - \frac{g}{2\rho_o} (h + \zeta) \frac{\partial \rho}{\partial y}$$

Semi-implicit FD: Algebraic Eq. of $\zeta_{i,j}^{n+1}, V_{i,j+1/2}^{n+1}, \zeta_{i,j+1}^{n+1}$

Substituting the momentum Equations into

Continuity Eq.:
$$\frac{\partial \zeta}{\partial t} + \frac{\partial [(h + \zeta)U]}{\partial x} + \frac{\partial [(h + \zeta)V]}{\partial y} = 0$$

$$(1 + A_{i+1,j} + B_{i-1,j} + C_{i,j+1} + D_{i,j-1})\zeta_{i,j}^{n+1} - A_{i+1,j}\zeta_{i+1,j}^{n+1} - B_{i-1,j}\zeta_{i-1,j}^{n+1} - C_{i,j+1}\zeta_{i,j+1}^{n+1} - D_{i,j-1}\zeta_{i,j-1}^{n+1} = E_{i,j}^n$$

With all coefficients are positive.

The governing matrix equation is symmetric, diagonally dominant, and positive definite. Numerical solution is achieved by a preconditioned conjugate gradient method.

Some Numerical Properties

- **Convective terms- Eulerian-Lagrangian method**
- **Gravity wave terms - unconditionally stable**
- **Discretized equation - properly accounts for positive and zero depths**
- **Wetting and drying of cells are treated correctly**
- **TRIM2D successfully implemented to reproduce sharp hydrographs of riverine flows and for estuaries**
- **The model is robust and efficient**

TRIM_2D: Extensive applications in San Francisco Bay

Cheng, R. T., V. Casulli, and J. W. Gartner, 1993, Tidal, residual, intertidal mudflat (TRIM) model and its applications to San Francisco Bay, California, Estuarine, Coastal, and Shelf Science, Vol. 36, p. 235-280.

What does TRIM model stand for?

TRIM stands for **T**idal, **R**esidual, **I**nter-tidal **M**udflat

TRIM also implies **simple and elegant** in numerical algorithm and model code, a goal that we are striving for!

From TRIM Series of Models to UnTRIM

Systematic Development of TRIM Models:

TRIM_3D: Applications in San Francisco Bay and others

Casulli, V. and R. T. Cheng, 1992, Inter. J. for Numer. Methods in Fluids

Casulli, V. and E. Cattani, 1994, Comput. Math. Appl., Stability, accuracy and efficiency analysis of TRIM_3D, θ -method for time-difference

Cheng, R. T. and V. Casulli, 1996, Modeling the Periodic Stratification and Gravitational Circulation in San Francisco Bay, ECM-4.

TRIM_3D: Non-hydrostatic

Casulli, V. and G. S. Stelling, 1996, ECM-4

Casulli, V. and G. S. Stelling, 1998, ASCE, J. of Hydr. Eng

UnTRIM model:

Casulli, V. and P. Zanolli, 1998, A Three-dimensional Semi-implicit Algorithm for Environmental Flows on Unstructured Grids, Proc. of Conf. On Num. Methods for Fluid Dynamics, University of Oxford.

Extension to Unstructured Grid Model -- UnTRIM

TRIM Modeling Philosophy:

1. Semi-implicit Finite-Difference Methods
2. Θ -Method for time difference
3. Solutions in **Physical Space**, regular mesh, no coordinate transformations in x-, y-, or z-directions
4. In complicated domain, refine grid resolution if necessary
5. Pursue computational efficiency and robustness

UnTRIM (Unstructured Grid TRIM model) follows the SAME TRIM modeling philosophy, except the finite-difference cells are boundary fitting unstructured polygons!

Summary of the UnTRIM Model:

Governing equations (Hydrostatic Assumption)

Continuity and Free-surface Equations

$$\text{Div}(\vec{U}) = 0$$

Incompressibility

$$\frac{\partial \zeta}{\partial t} + \nabla \bullet \left[\int_{-h}^{\zeta} \vec{V} dz \right] = 0$$

Free-surface equation

Horizontal Momentum Equation in \vec{N}_j direction for velocity V_j

$$\frac{DV_j}{Dt} - f(\nabla \times \vec{V}) \bullet \vec{N}_j = \frac{\partial}{\partial z} (\mathbf{v}_v \frac{\partial}{\partial z} V_j) + \mathbf{v}_h \nabla^2 V_j - g \frac{\partial \zeta}{\partial N_j} - \frac{g}{\rho_0} \frac{\partial}{\partial N_j} \int_z^{\zeta} (\rho - \rho_0) dz'$$

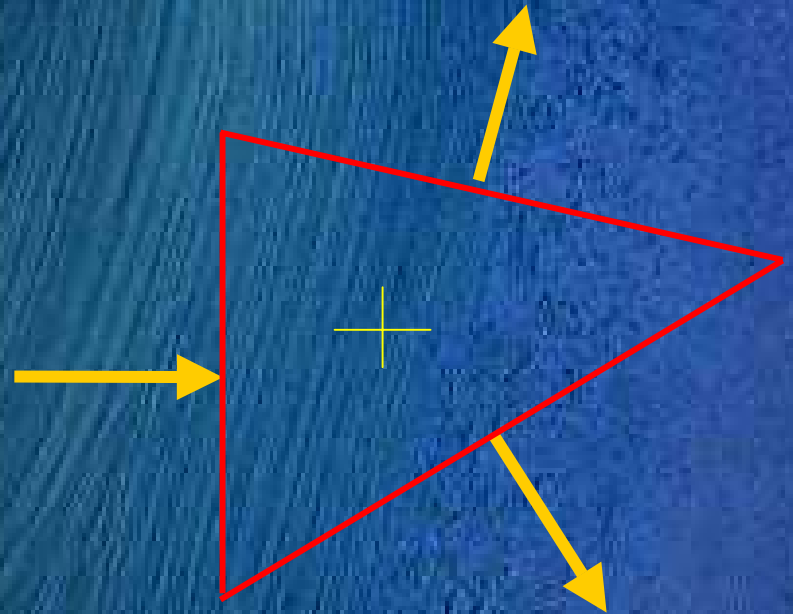
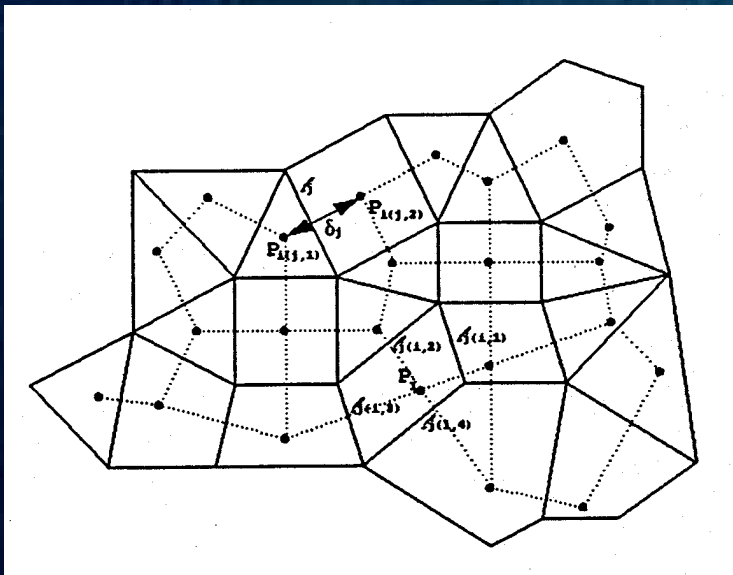
where $\nabla \times ()$ is cross product, $\nabla \bullet ()$ is inner product, $\nabla^2 ()$ is the Laplacian, and \vec{V} is the velocity in the horizontal plane.

Transport Equations

$$\frac{D}{Dt} C_j = \frac{\partial}{\partial z} (K_v \frac{\partial}{\partial z} C_j) + K_h \nabla^2 C_j \quad j = 1, 2, 3, \dots \text{ Lagged one time-step}$$

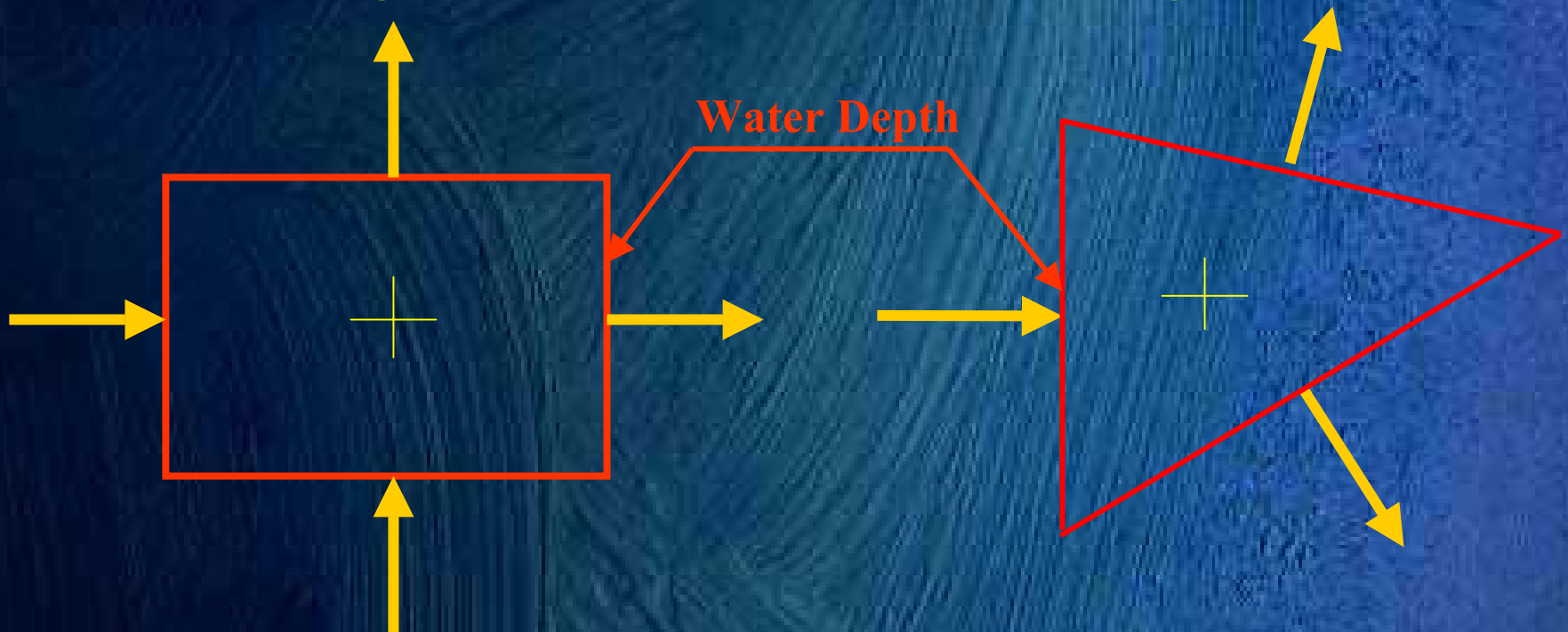
And an equation of State

1. Semi-implicit finite-difference of momentum Eq. in the normal direction to each face is applied!
2. Applied the Finite-Volume integration of the free surface equation!
Local and global conservation of volume is guaranteed!



3. The resultant matrix equation determines the water surface elevation for the entire field.

1. Semi-implicit finite-difference of momentum Eq. in the normal direction to each face is applied!
2. Applied the **Finite-Volume** integration of the free surface equation!
Local and global conservation of volume is guaranteed!



3. The resultant matrix equation determines the water surface elevation for the entire field.

Summary of Numerical Algorithm

Momentum Equation in \vec{N}_j direction for velocity V_j relates V_j and ζ (left) and ζ (right) on each face of a polygon

Continuity and Free-surface Equations

$$\text{Div}(\vec{U}) = 0$$

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot \left[\int_{-h}^{\zeta} \vec{V} dz \right] = 0 \quad \Rightarrow \quad \frac{\partial \zeta}{\partial t} + \oint \left(\int_{-h}^{\zeta} \vec{V} dz \right) \cdot d\vec{s} = 0$$

Finite Volume integration over each polygon \Rightarrow

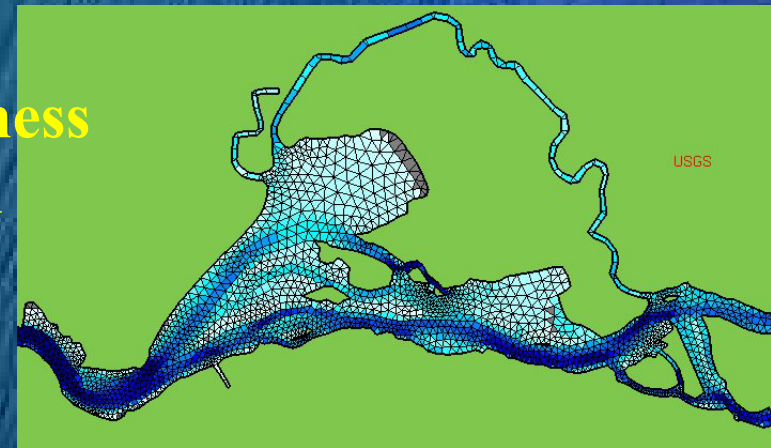
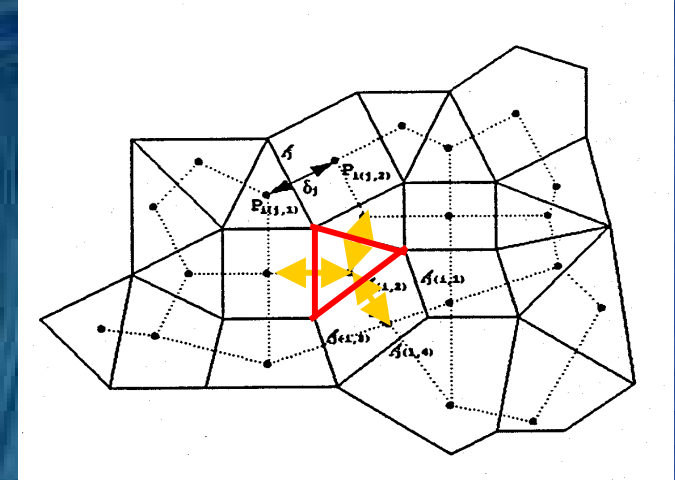
V 's are eliminated giving a Matrix Eq. for ζ

The continuity equation and the momentum equations are truly coupled in the solution. **No mode splitting is used!**

Issues of unstructured grids

User must define:

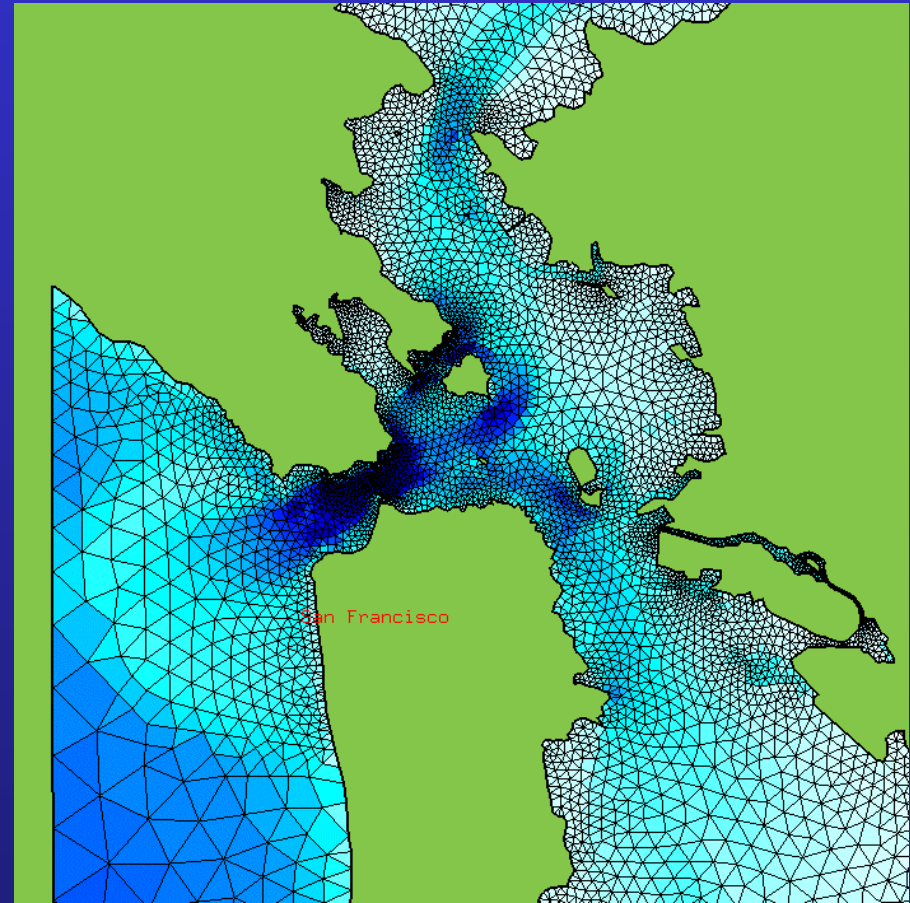
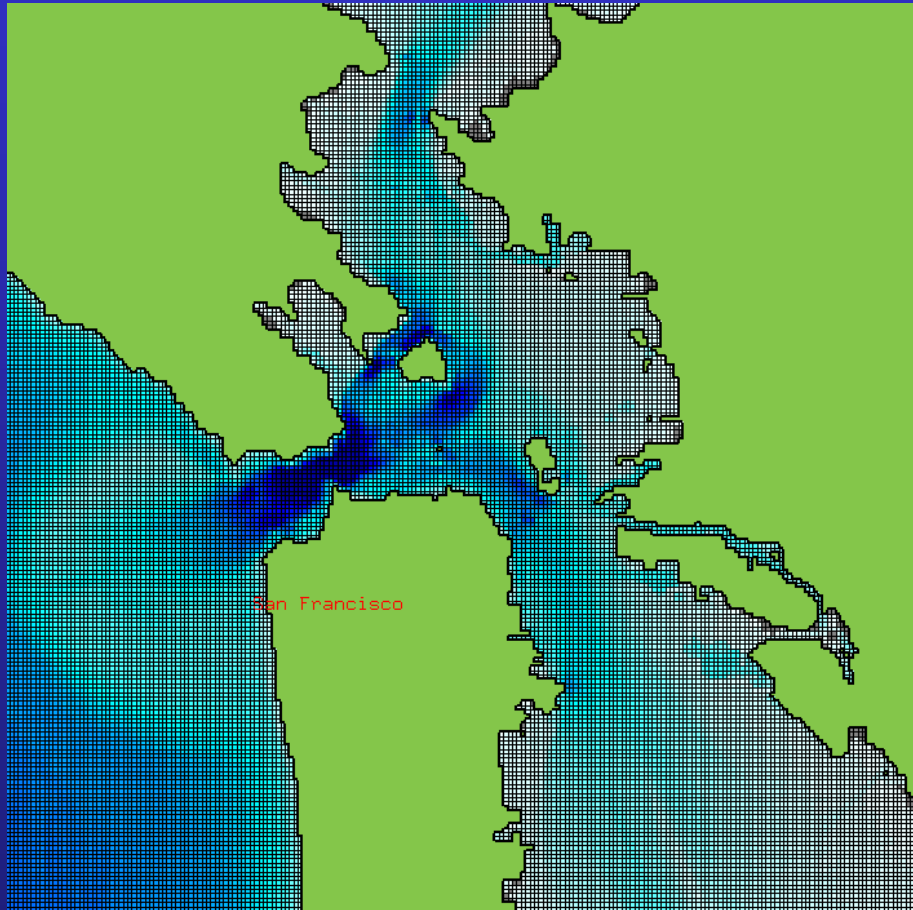
1. Number and locations of nodes
2. Polygon number and its relation with nodes (connectivity)
3. Each side is numbered, left and right polygons are defined (connectivity)
4. Center coordinates of each polygon
5. Vertical layers are of constant thickness (variable in z) except the bottom and free-surface; a stack of prisms
6. Water depth and normal velocity are defined on the sides
7. Water elevation is defined at the center of the polygon



(All Rectangles)

San Francisco Bay

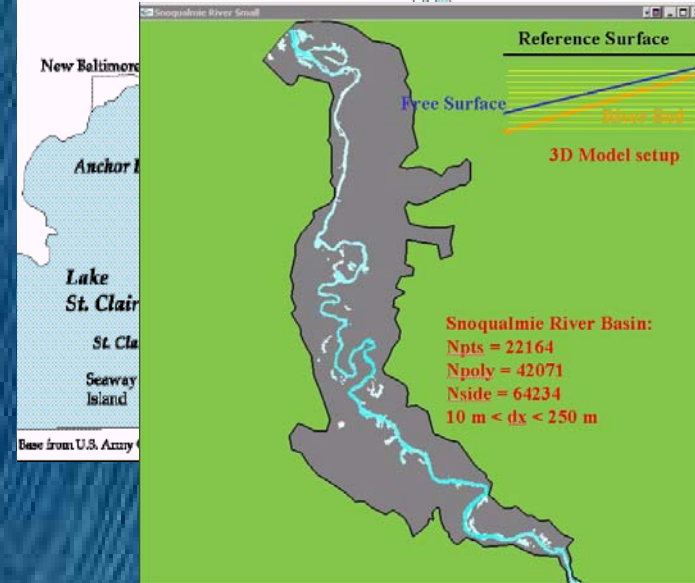
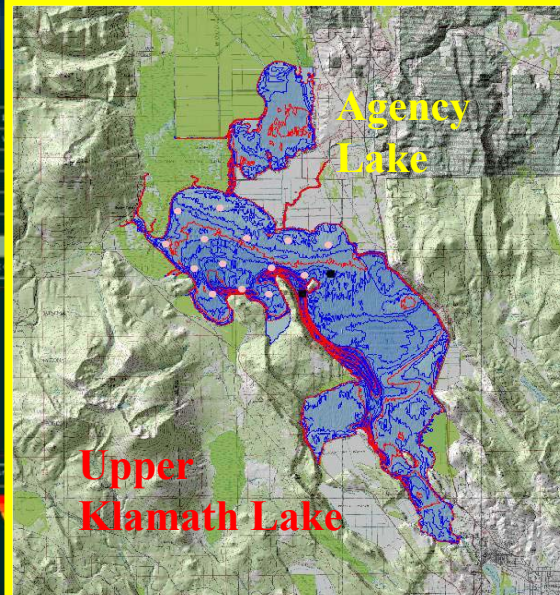
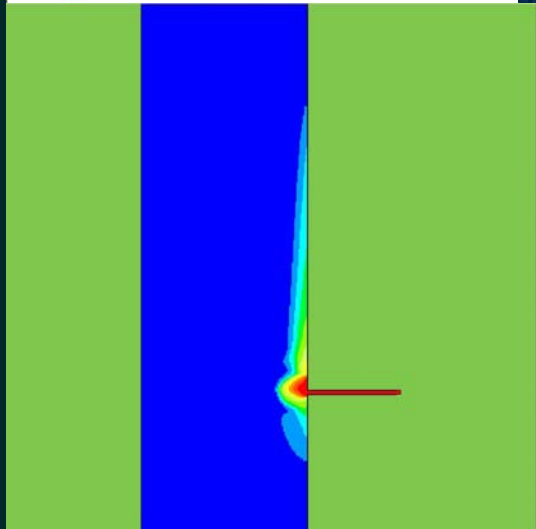
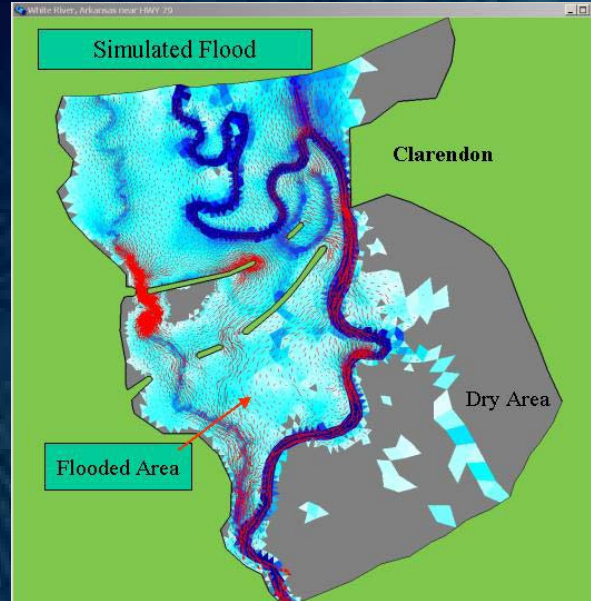
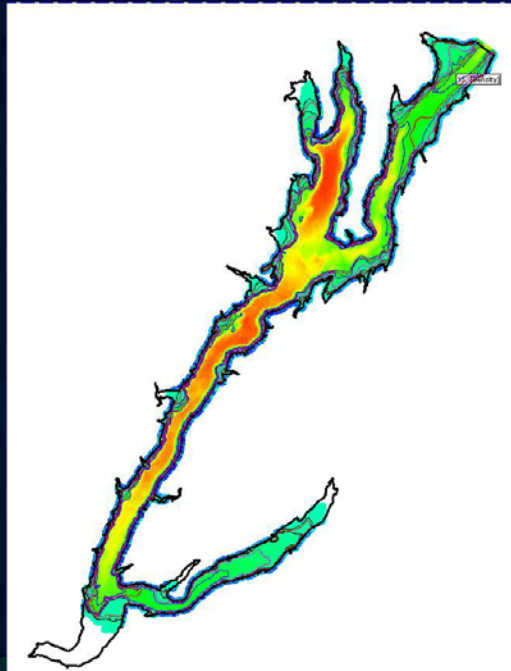
(Mixed Polygons)



48506 nodes, 45841 polygons
94374 sides on the top layer
42 layers, **1,160 K faces**, $\Delta t = 180$
(R= simulation/CPU = 17.7)
on 2.2 GHz PC

12682 nodes, 20126 polygons
32827 sides on the top layer
42 layers, **295 K faces**, $\Delta t = 180$
(R= simulation/CPU = 70)
on 2.2 GHz PC

The UnTRIM Model has been applied to Rivers, Lakes and Estuaries



An UnTRIM application

Modeling Wind-Driven Circulation in Upper Klamath Lake

Ralph T. Cheng*

Jeffrey W. Gartner*

Tamara Wood**

***U. S. Geological Survey, Menlo Park, CA**

****U. S. Geological Survey, Portland, OR**

I. Background and Issues

II. ADCP Deployment and Results

III. Time-series of Wind Observations

IV. Wind-Driven Circulation

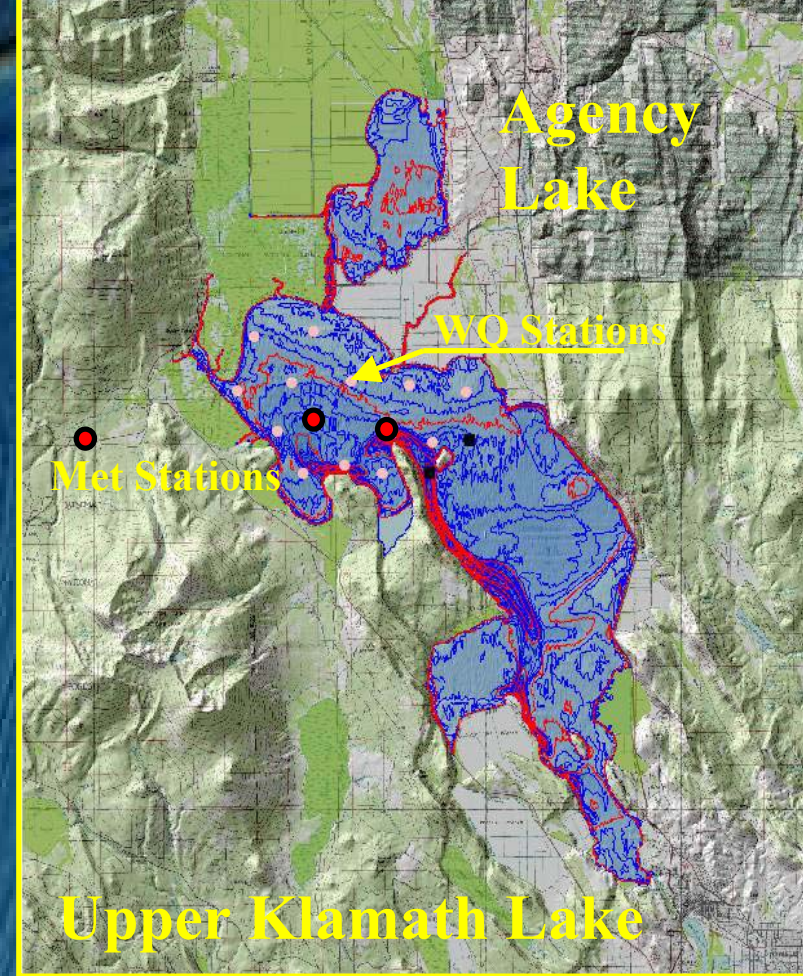
V. Reproducing ADCP Observations

VI. Analyze This and Analyze That

VII. Conclusion (Physics Rules!)

Background

- 120 mi² surface area
- Mean depth 8 ft
- Drains 3800 mi² phosphorus-rich volcanic soils
- Naturally eutrophic
- Wetlands drained and logging since 1900
- Presently hyper-eutrophic
- chlorophyll ~100-300 mg/L
- pH exceeding 9.5
- Ammonia ~ 500 mg/L
- DO approaching zero during a crash
- Massive annual cyanobacteria blooms

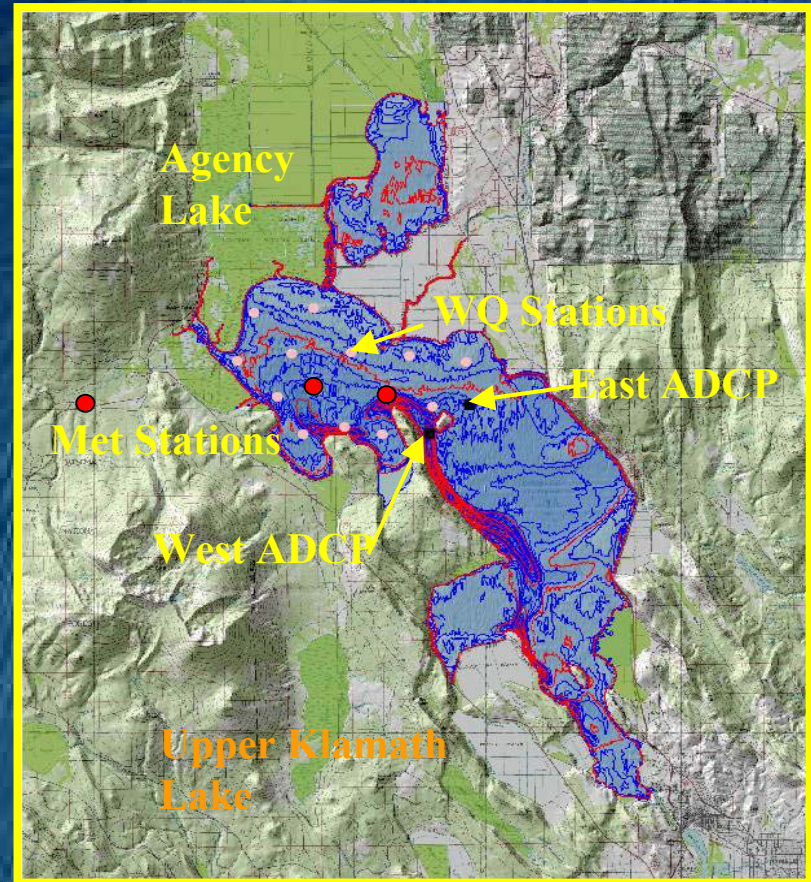


2002: USGS/BOR began a cooperative study of fish behavior in response to lake water quality conditions

Two main tasks:

- Tracking of tagged adult suckers using telemetry over northern 1/3 of UKL**
- A network of continuous WQ monitors that would provide a “map” of water quality**

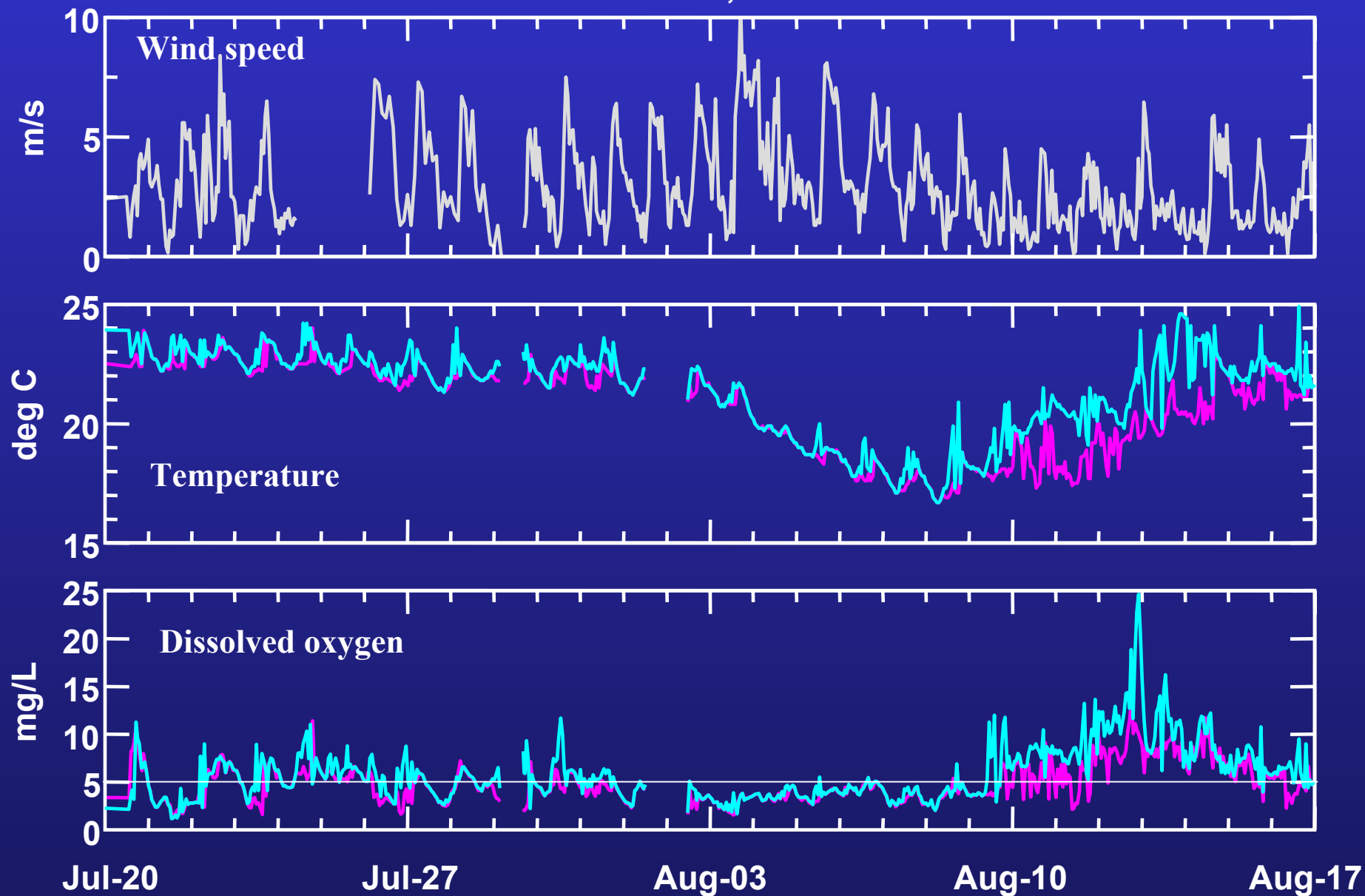
Wind Observations on the lake (Russ7)



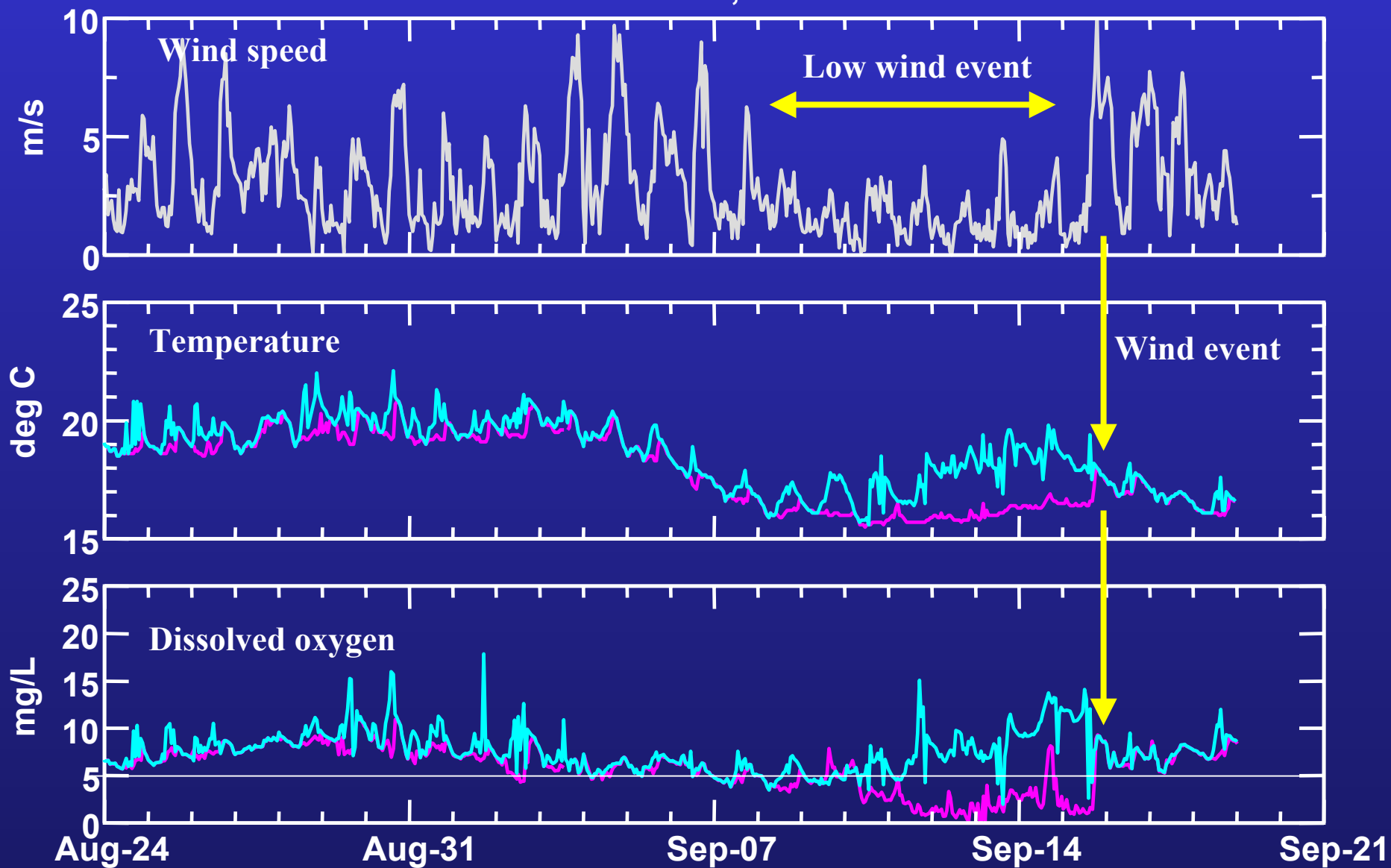
Issues with wind time-series:

1. Magnetic north
2. Data gaps or irregular time intervals

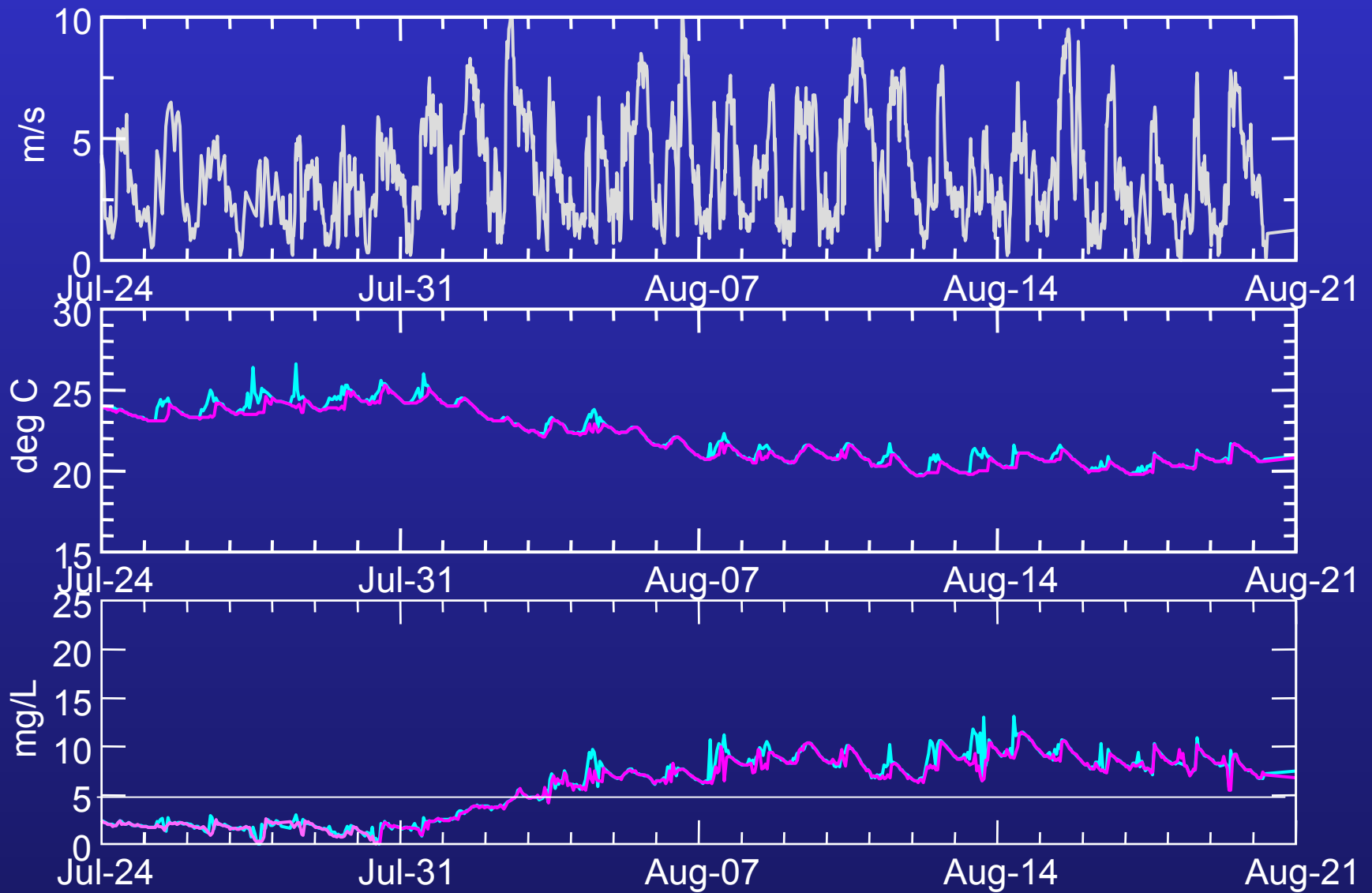
RUSS8, 2002



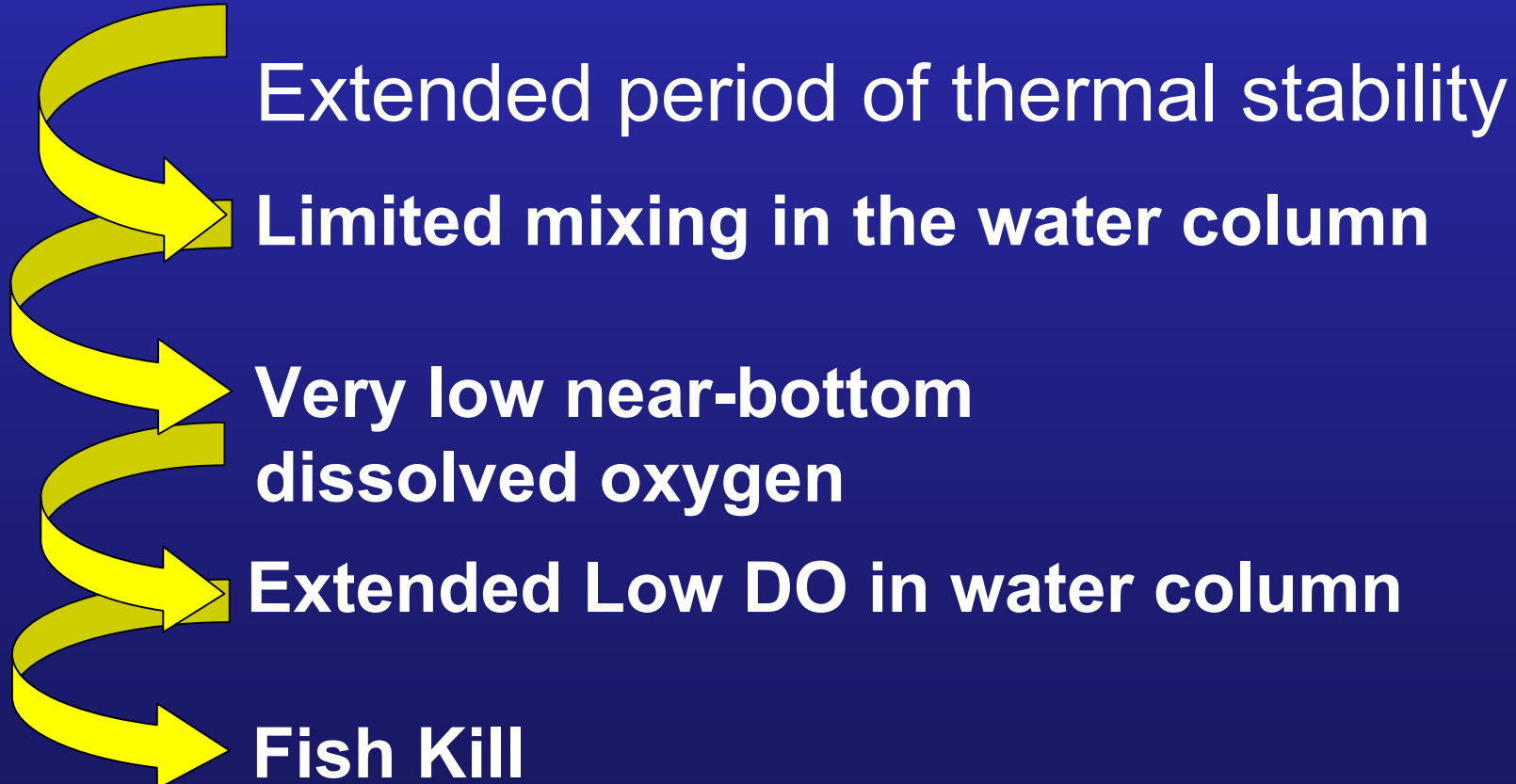
RUSS8, 2002



RUSS7, 2003



Sequence of Events Leading to a Fish Kill (prevailing wisdom):



Water Year 2003

40 km x 80 km

Agency Lake

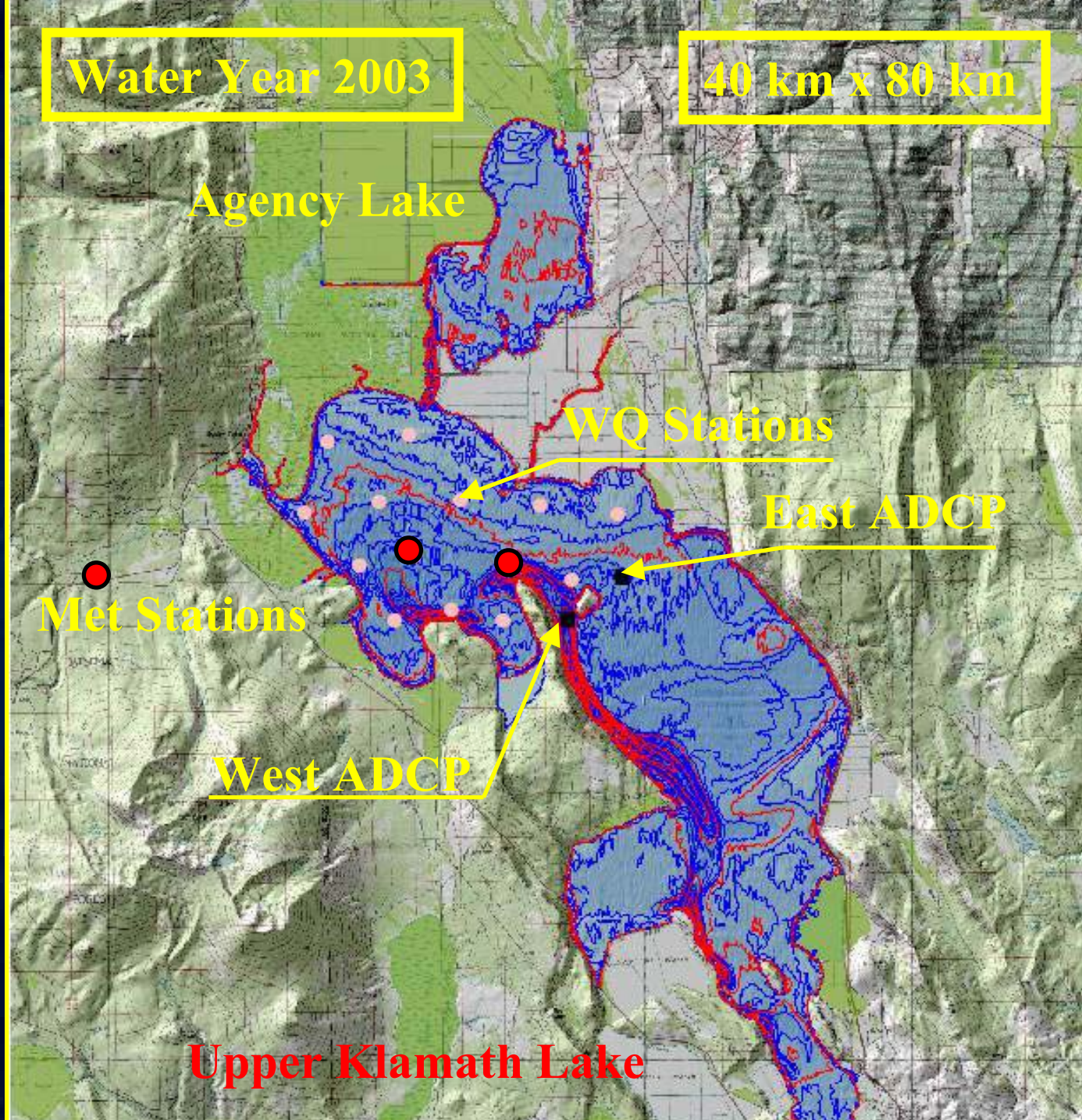
WQ Stations

East ADCP

Met Stations

West ADCP

Upper Klamath Lake



West ADCP Station:

Water depth ~ 8 m

Bin size = 0.2 m

Sampling rate = 30.0 min

Total bins = 34

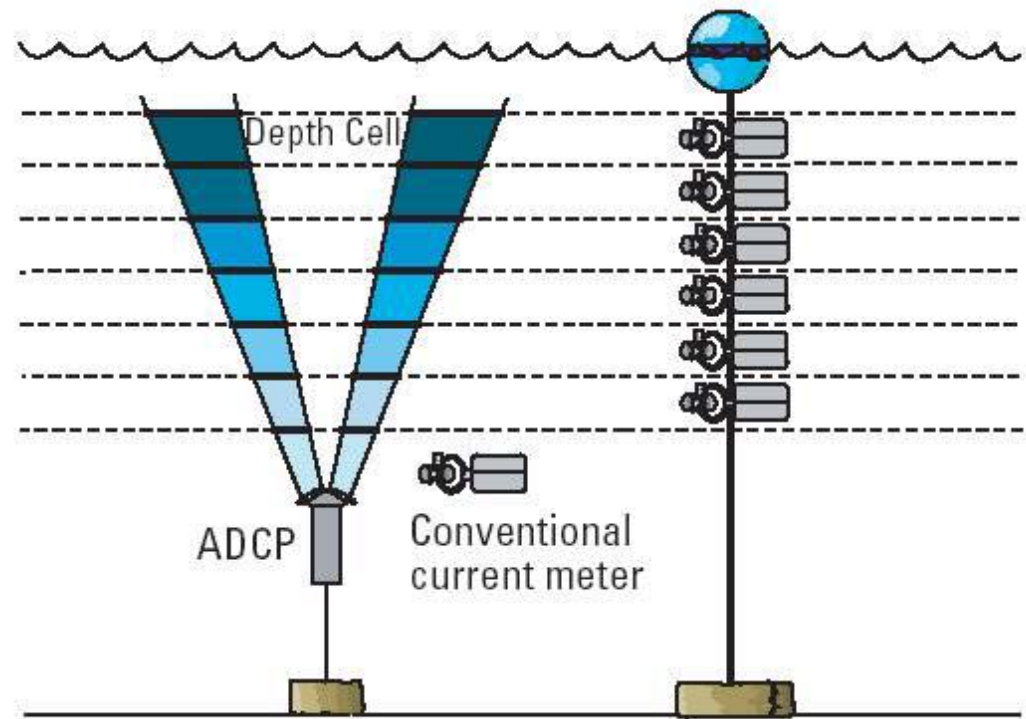
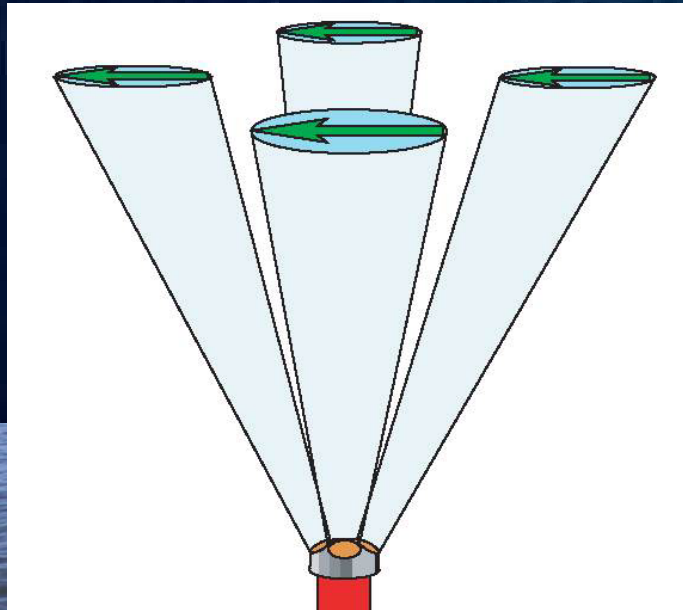


Figure 1.15. Analogy of a conventional current-meter string to an acoustic Doppler current profiler (ADCP) profile.

East ADCP Station:

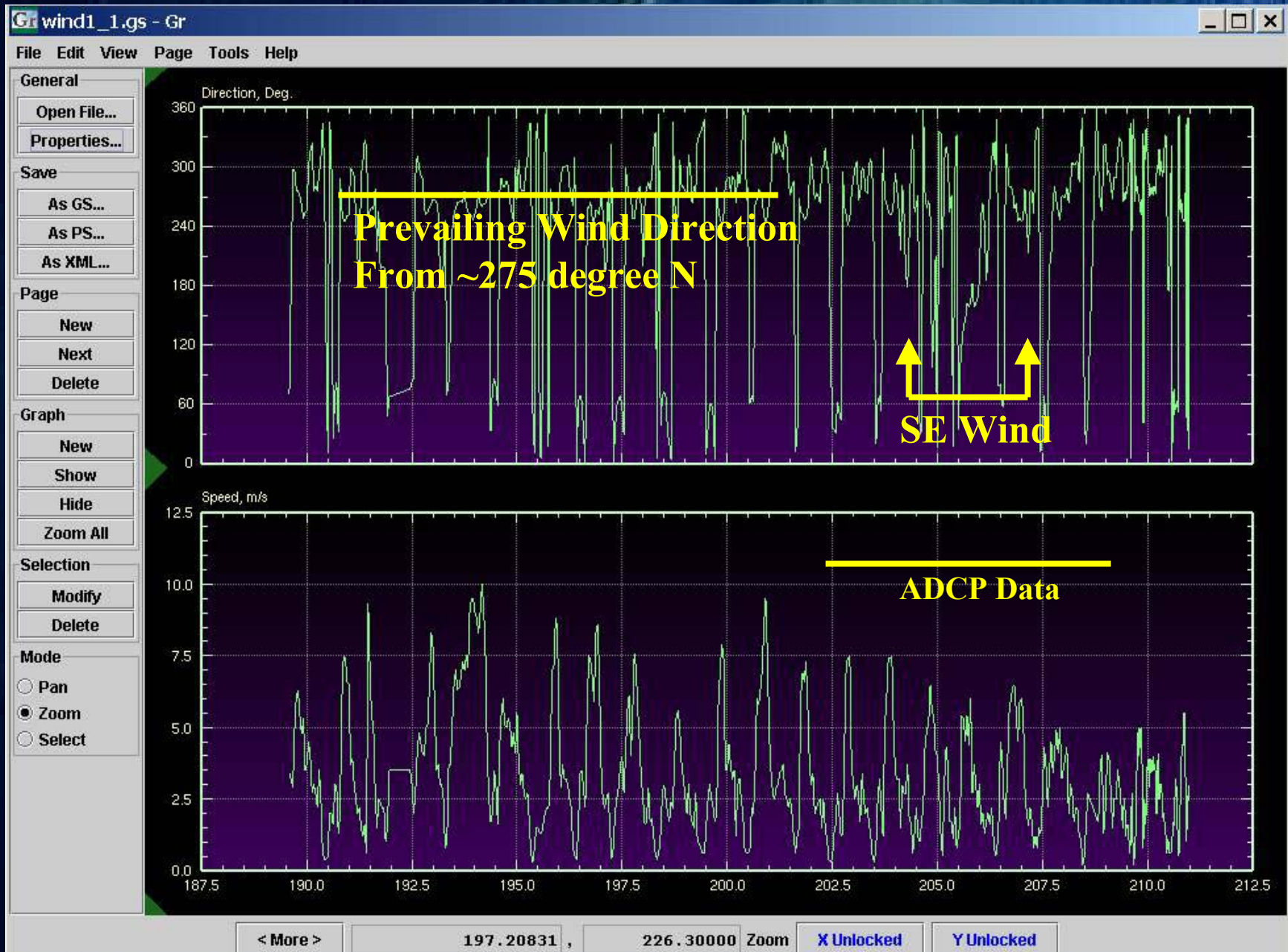
Water depth ~ 3.5 m

Bin size = 0.2 m

Sampling rate = 30.0 min

Total bins = 12

Wind Speed and Direction Time-Series



Filtered 3D ADCP Time-Series

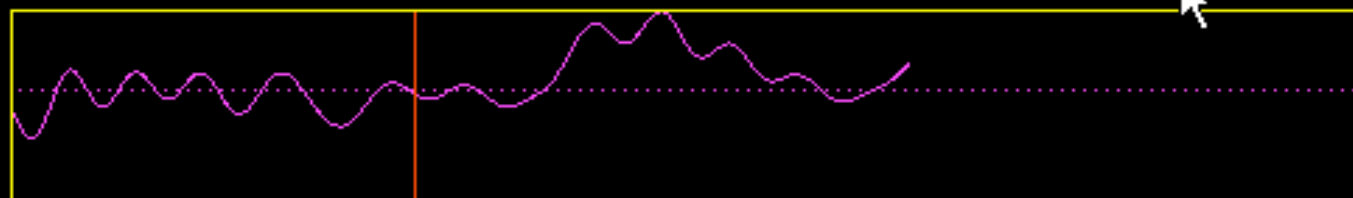
USGS Velocity Profile Viewer

07/21/03 22:30

Velocity in cm/s

UKL1A

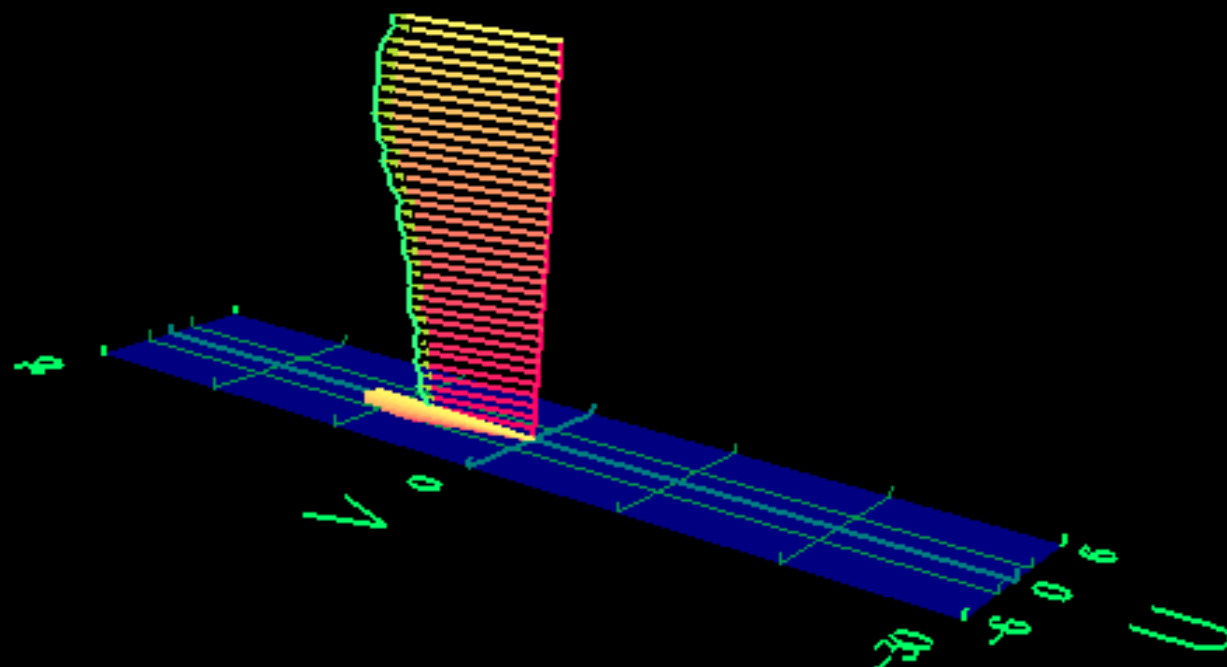
Prin Dir = None



1.1

Bin-
Averaged
U

-4.4



Rate = 1/2 (1/2)

Time Step 852 of 1217

General

Open File...

Properties...

Save

As GS...

As PS...

As XML...

Page

New

Next

Delete

Graph

New

Show

Hide

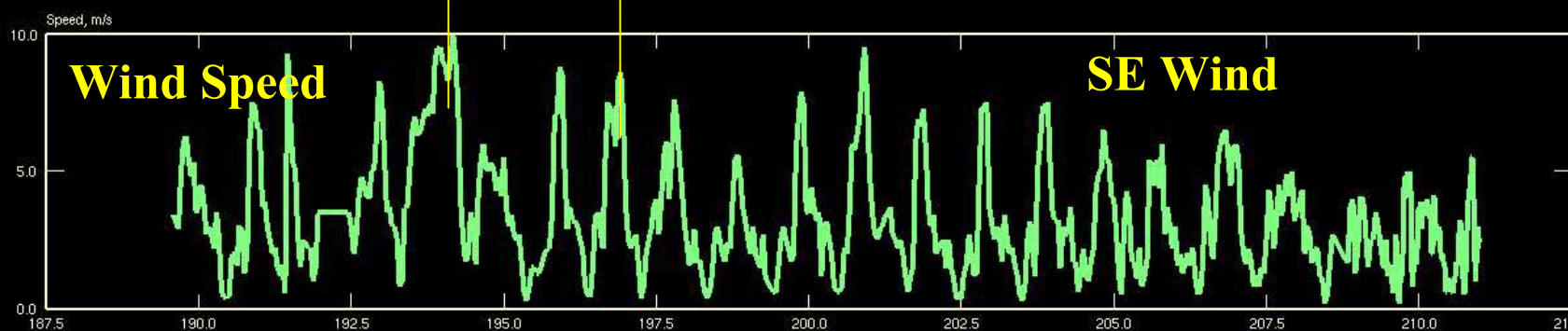
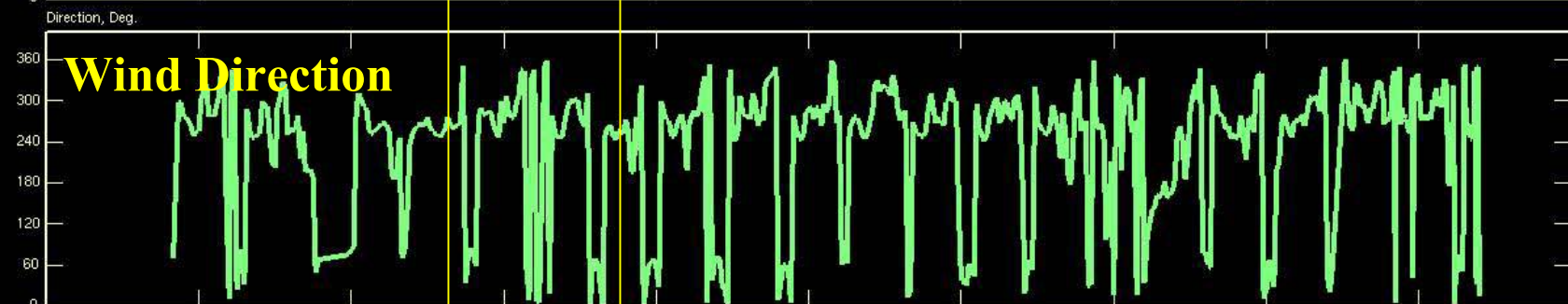
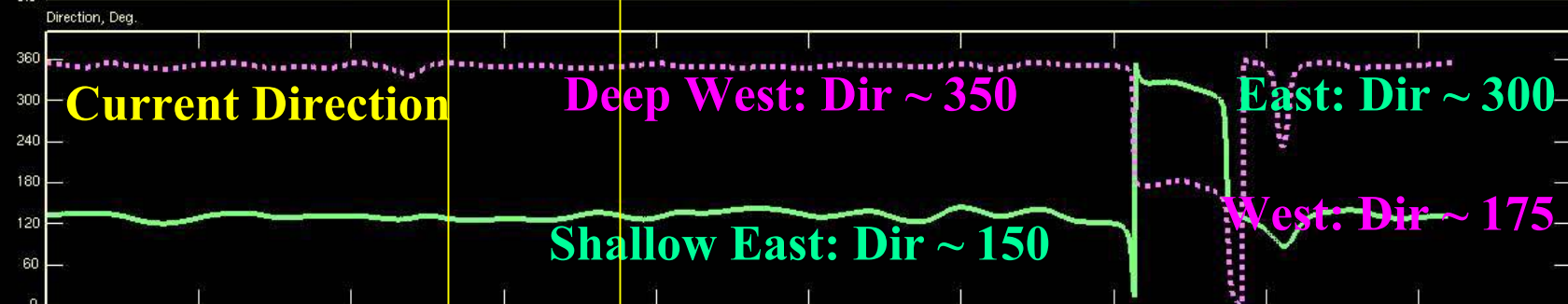
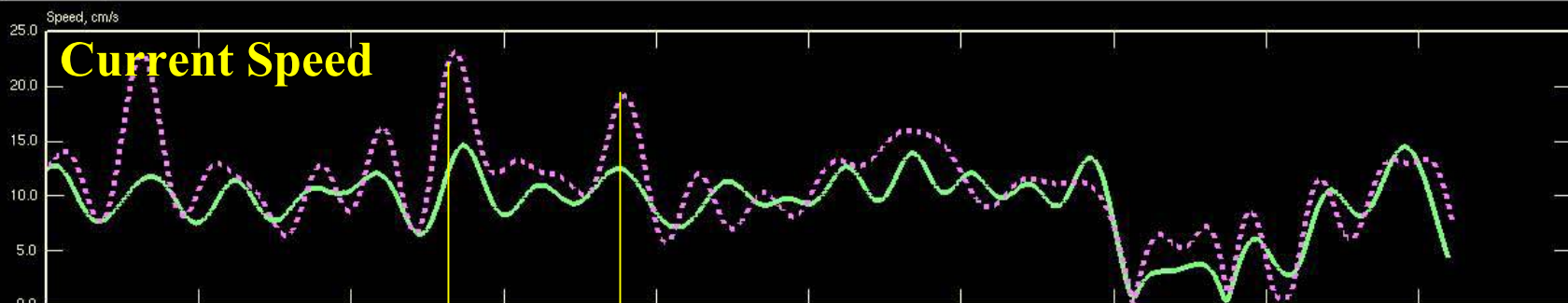
Zoom All

Selection

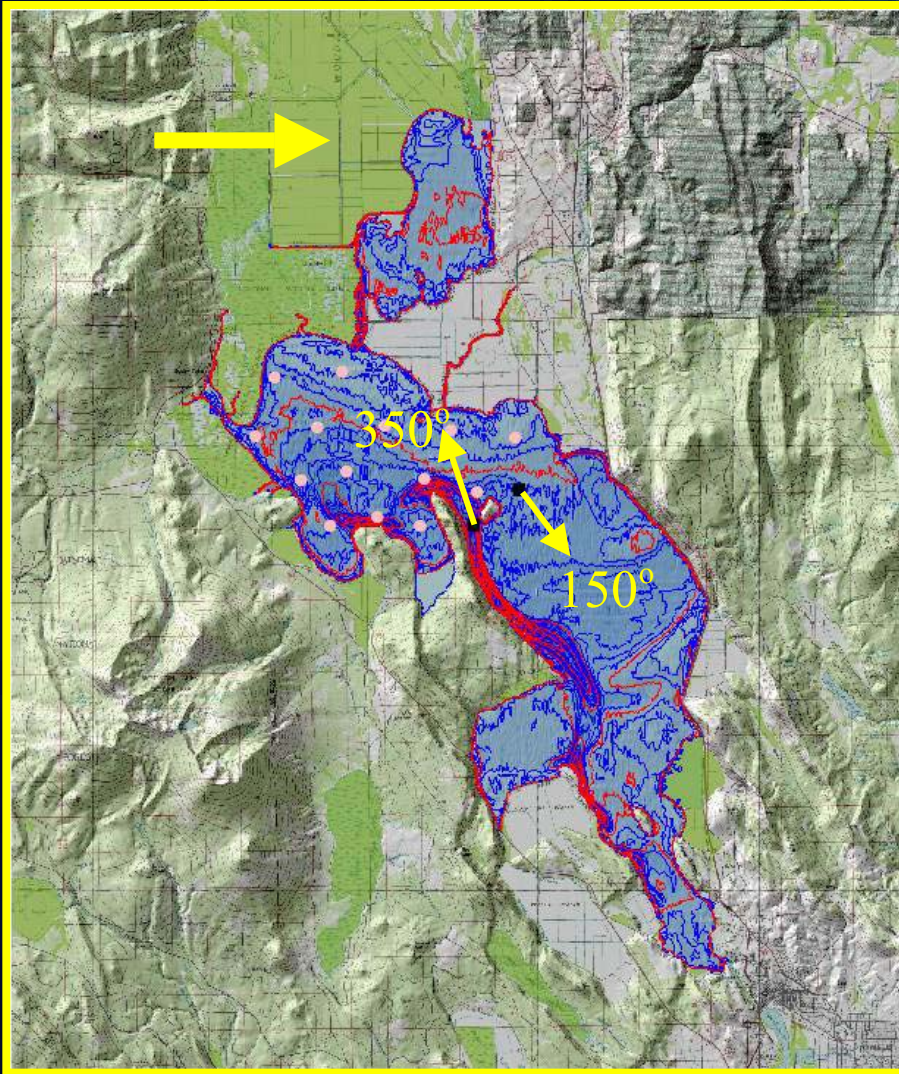
Modify

Delete

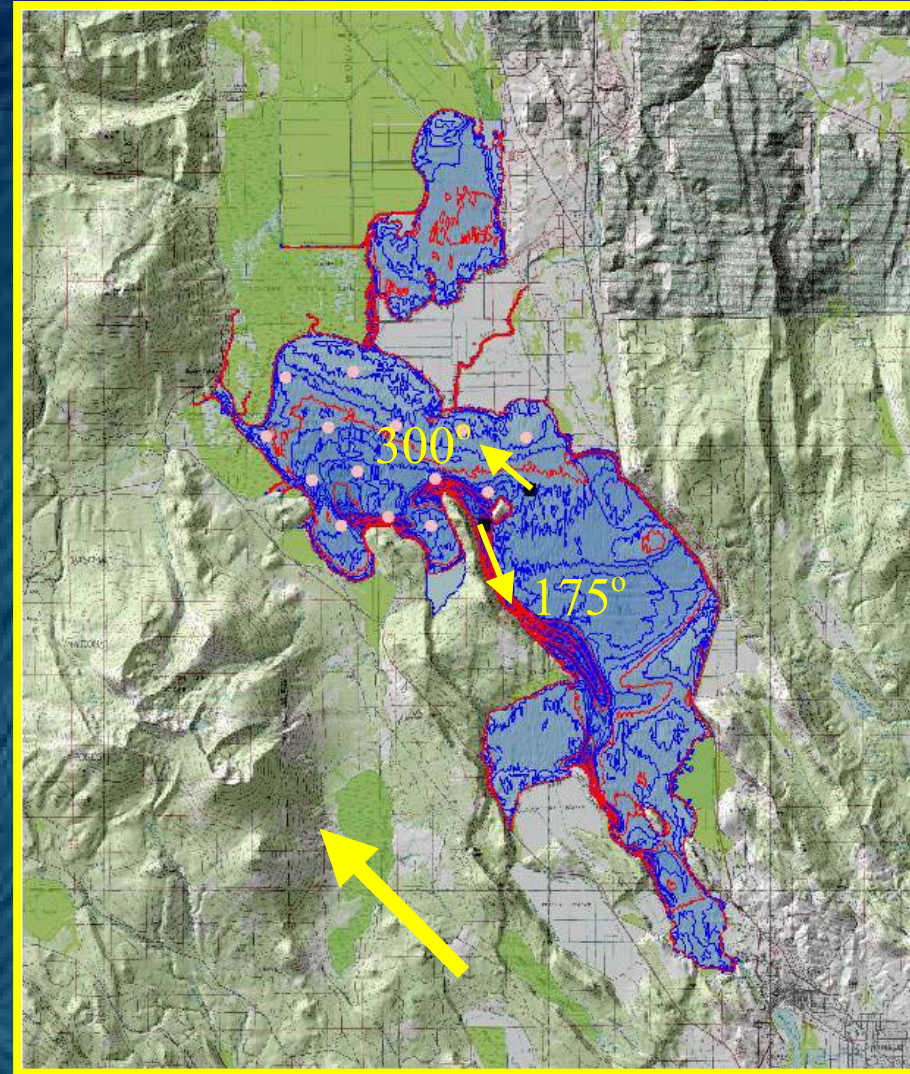
Mode

☒ Pan☐ Zoom☐ Select

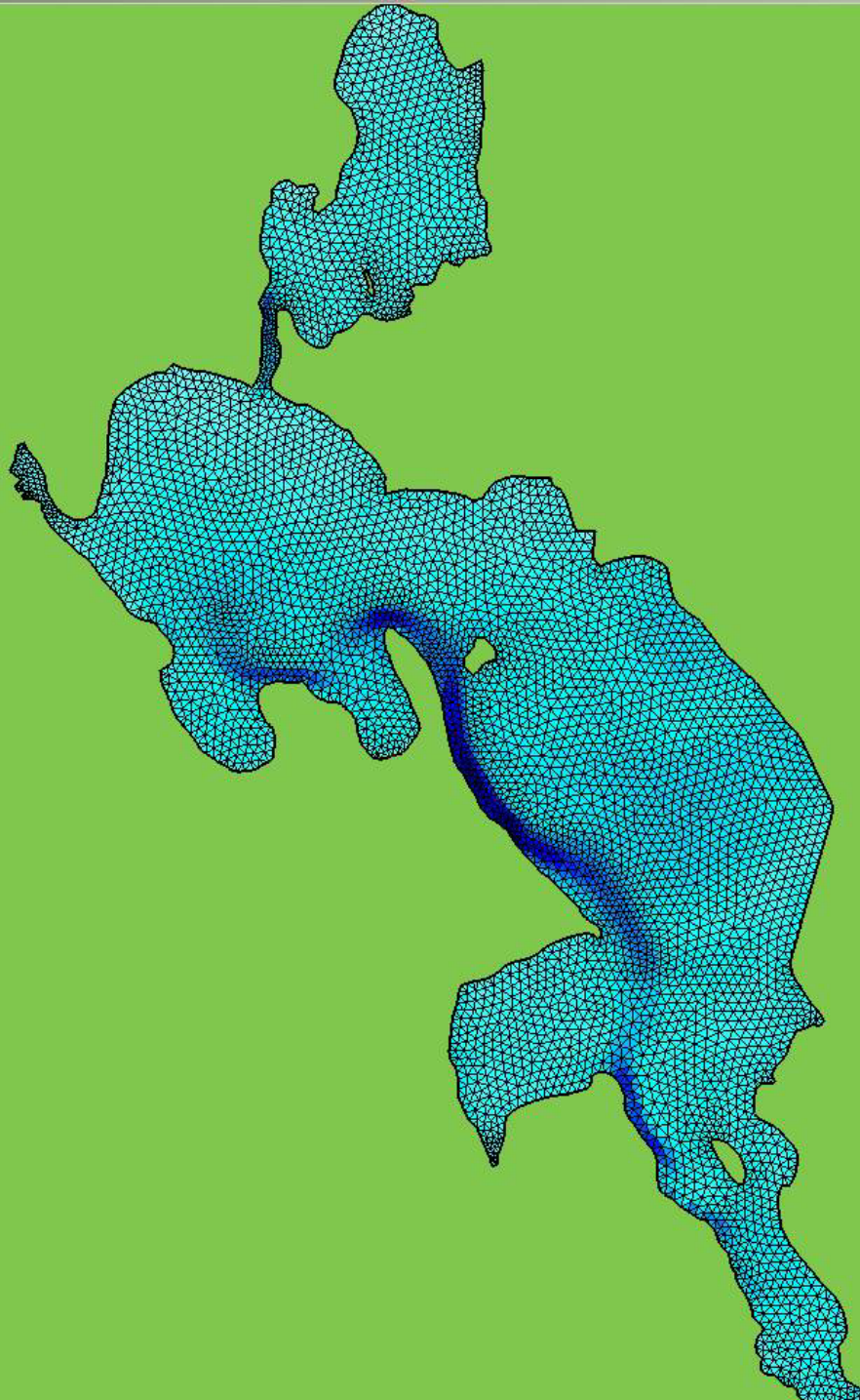
Synopsis of Wind-driven Circulation



Prevailing West Wind



SE Wind



**Unstructured
Grid Model:
Upper Klamath
Lake and Agency
Lake:**

$nv = 4712$

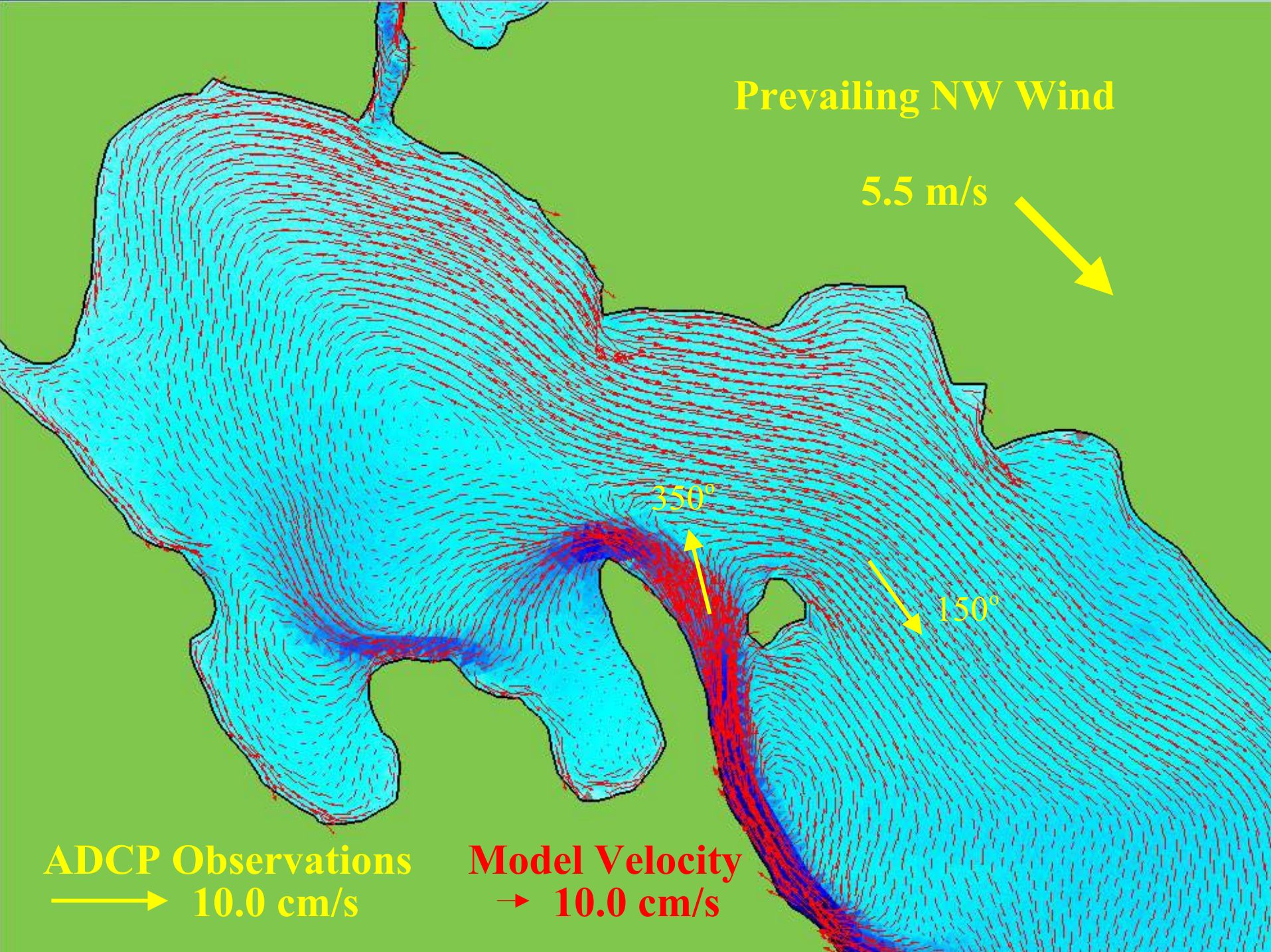
$ne = 8550$

$nk = 22$

$n3s = 82992$

**Side length
40 to 250 m**

**Grids are
boundary fitting
Fine resolution
grids for high
spatial variability.**



Prevailing NW Wind

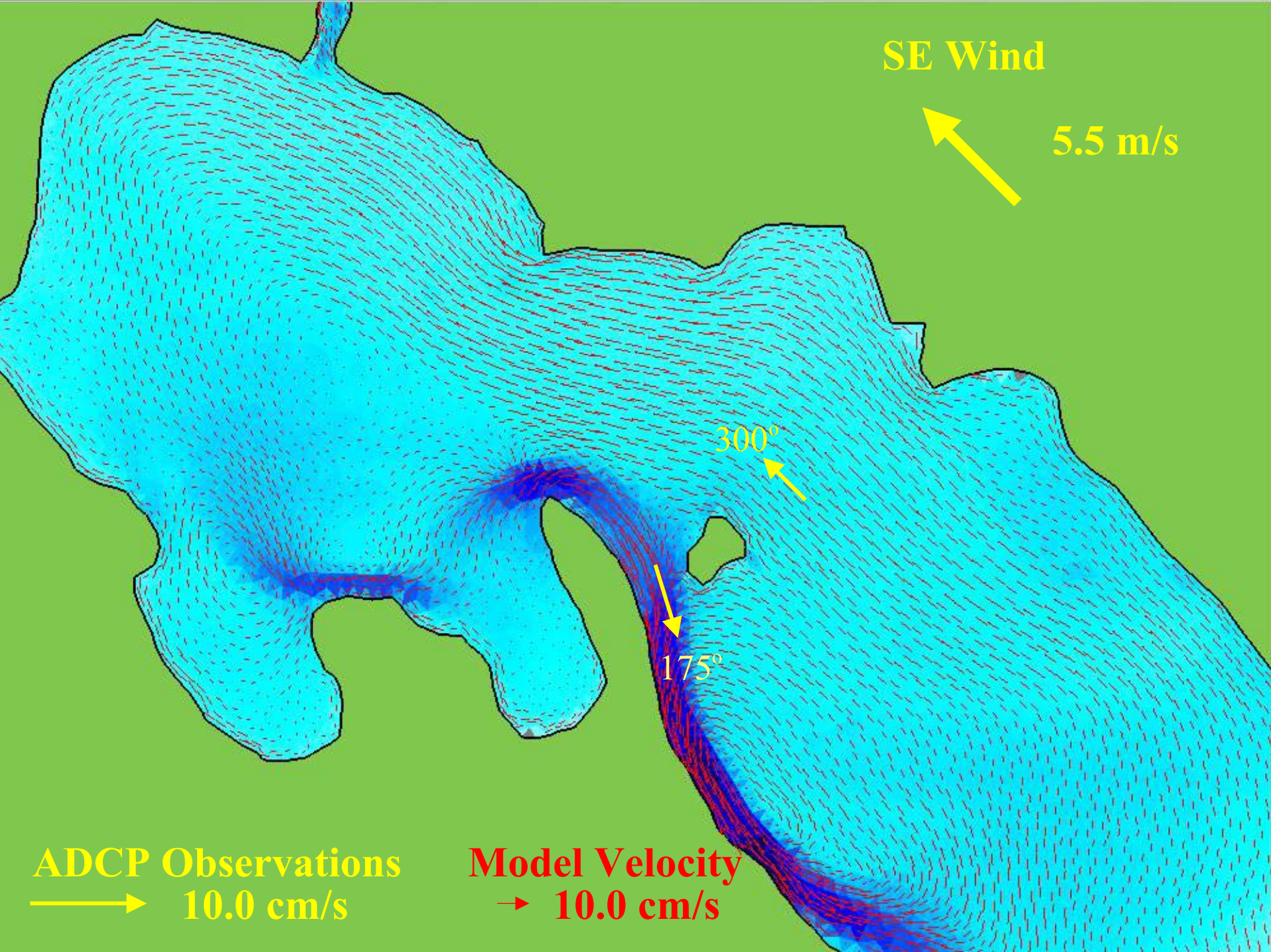
5.5 m/s

350°

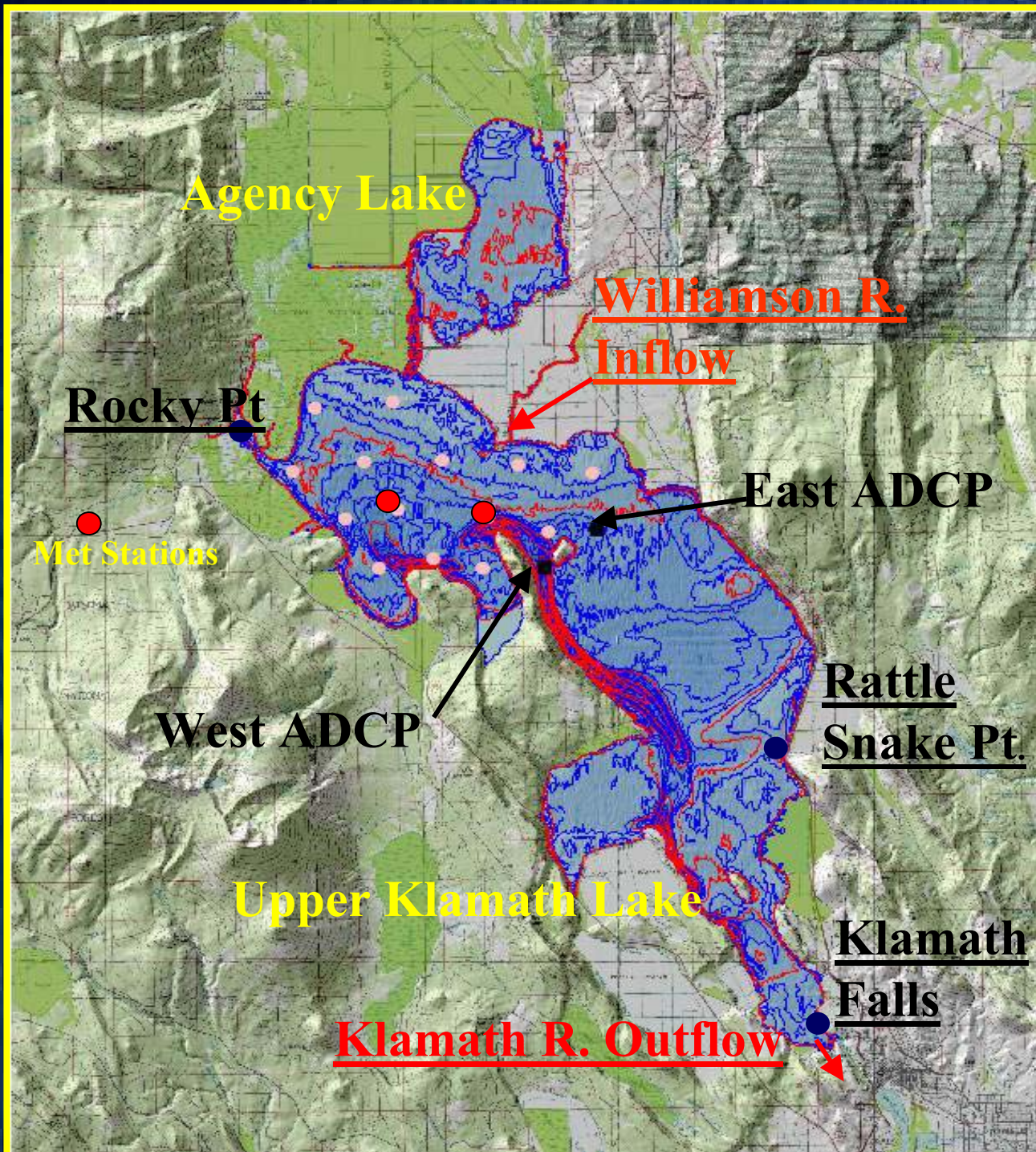
150°

ADCP Observations
→ **10.0 cm/s**

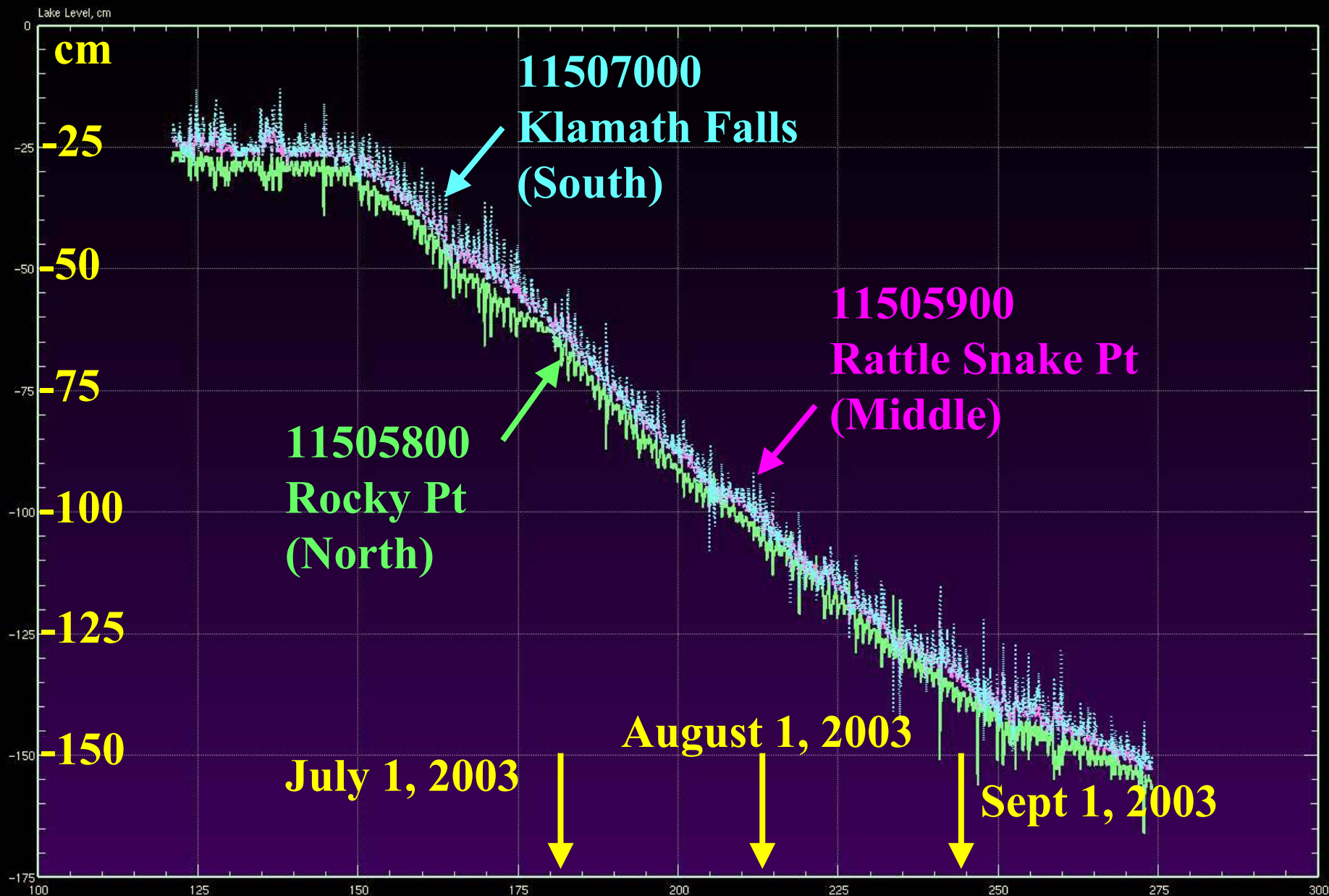
Model Velocity
→ **10.0 cm/s**



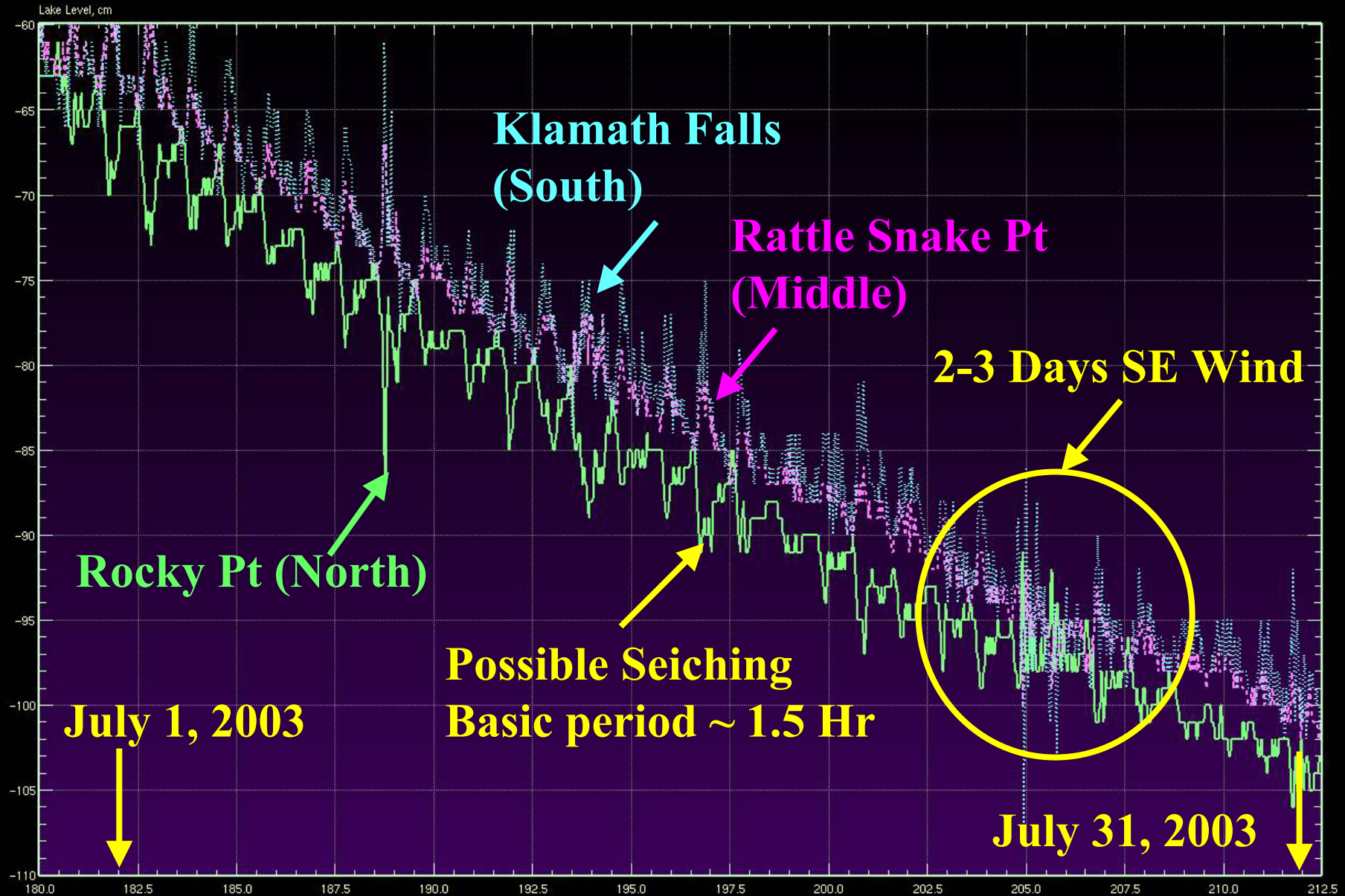
Field Data:
Observed Wind
Deep ADCP (West)
Shallow ADCP (East)
Williamson River Inflow
Klamath R. Outflow
Water levels at Rocky Pt
Rattle Snake Pt.
Klamath Falls



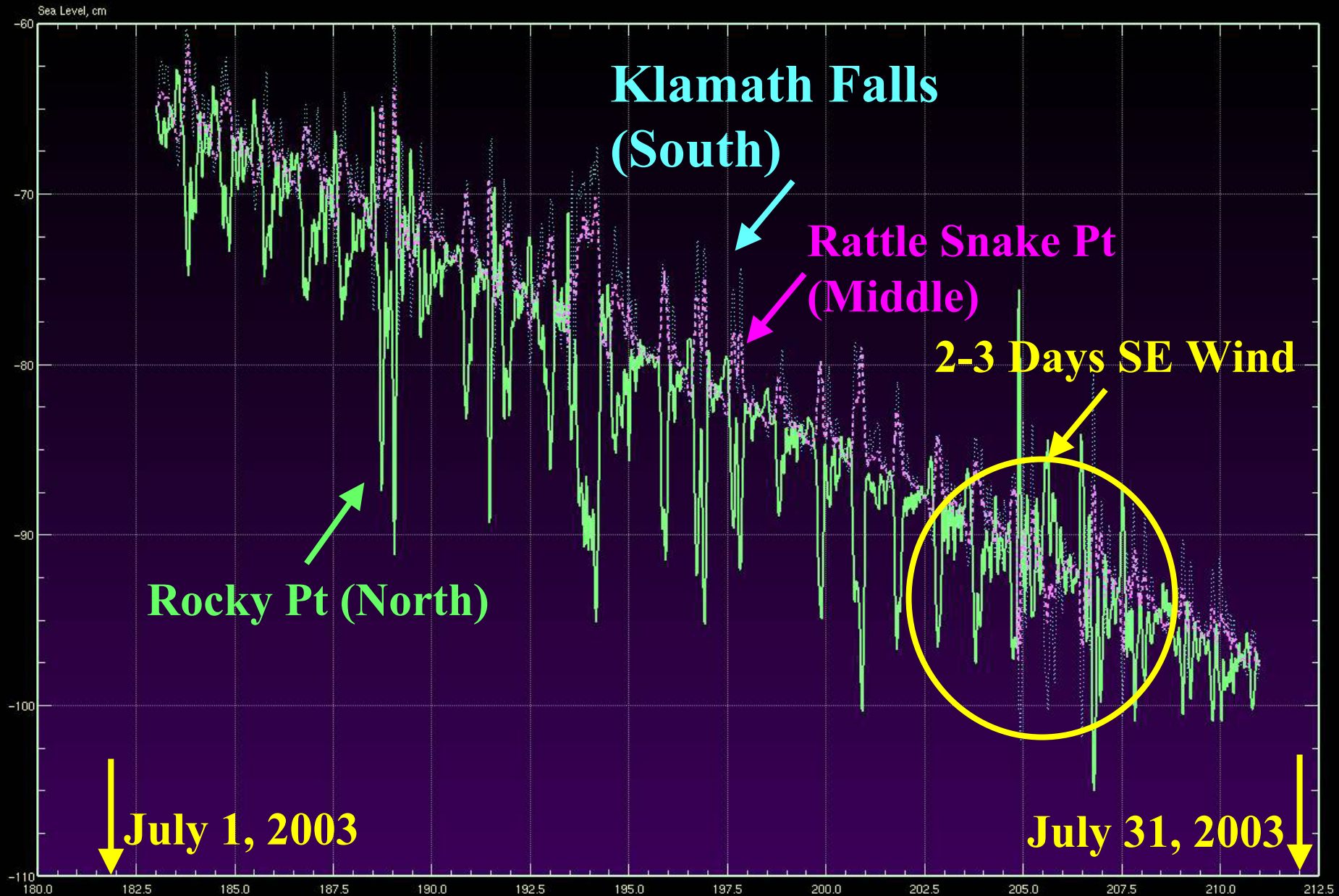
Water Level Observations Referenced to 4143 ft above sea-level



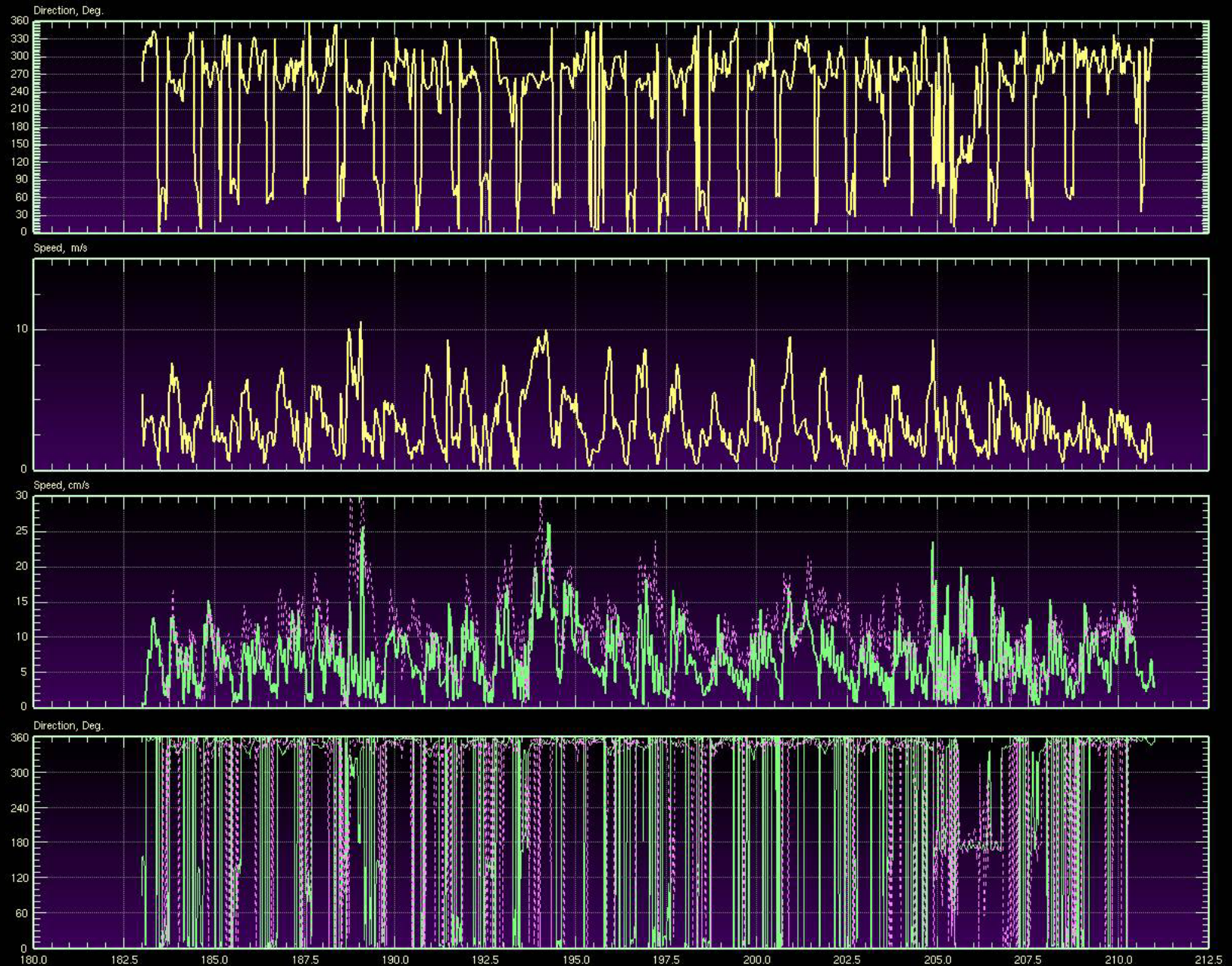
Water Level Observations Referenced to 4143 ft above sea-level



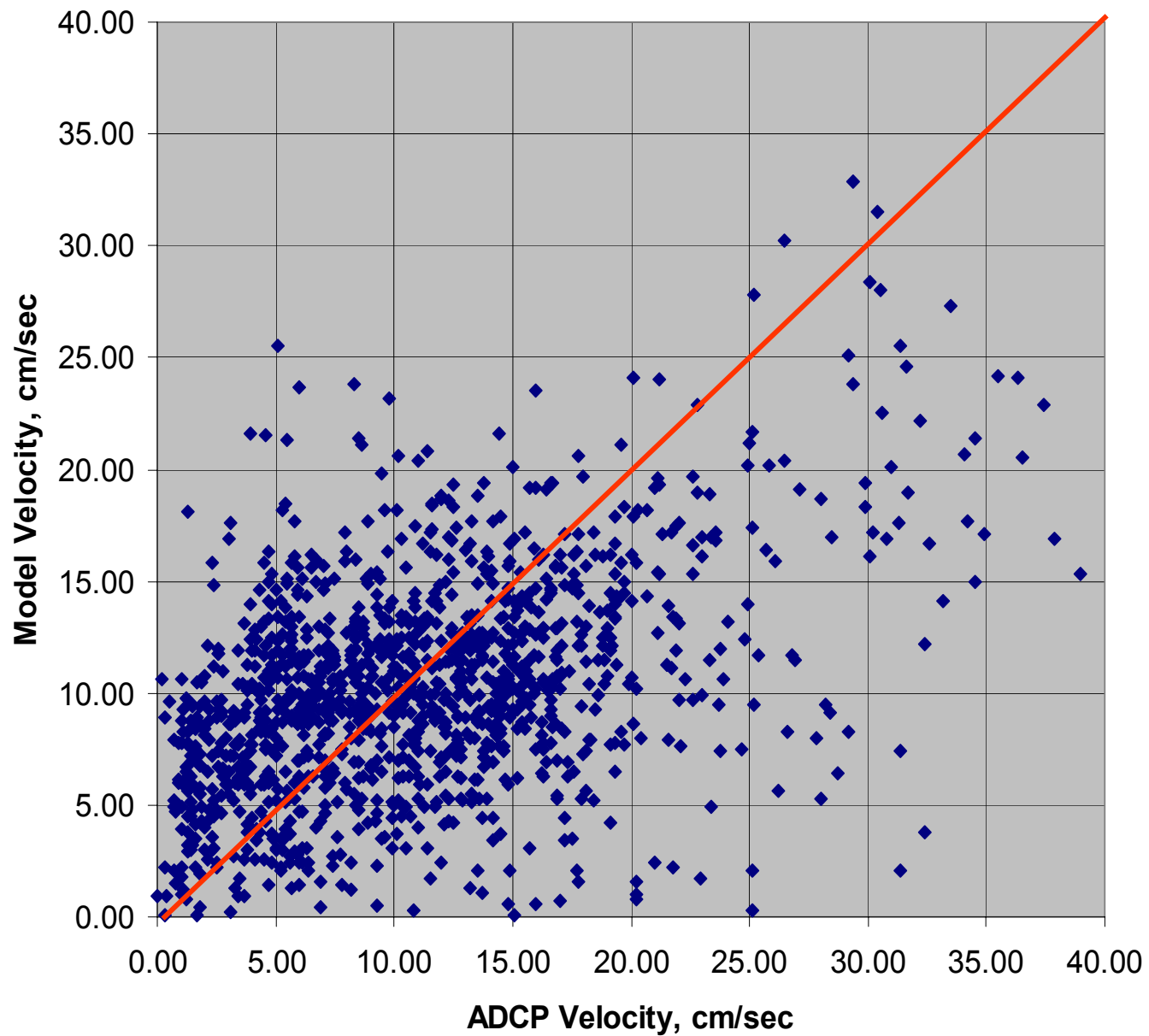
Model Simulated Water Level Variations



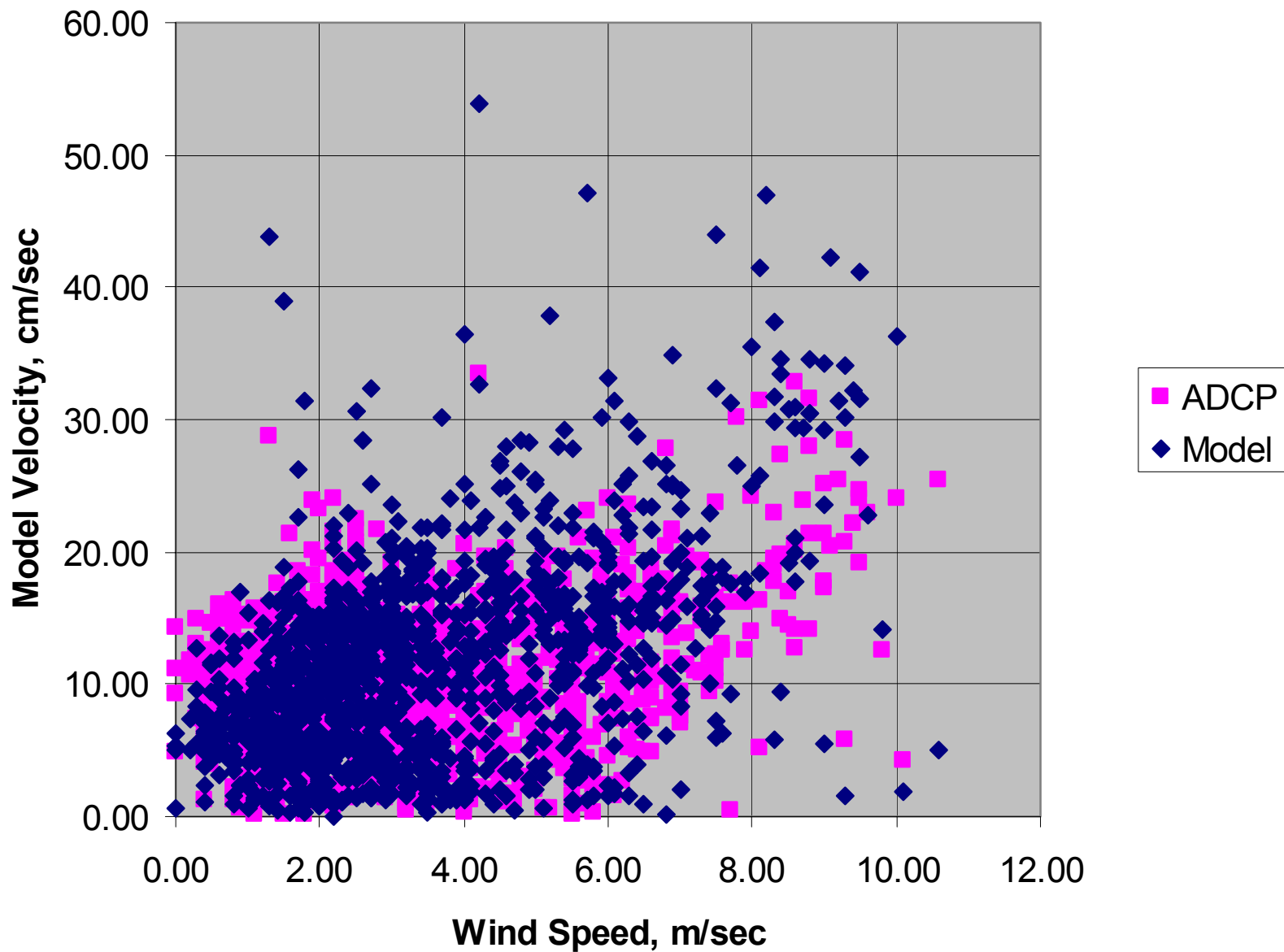
Model Results vs. ADCP Observations at Deep (West) Station



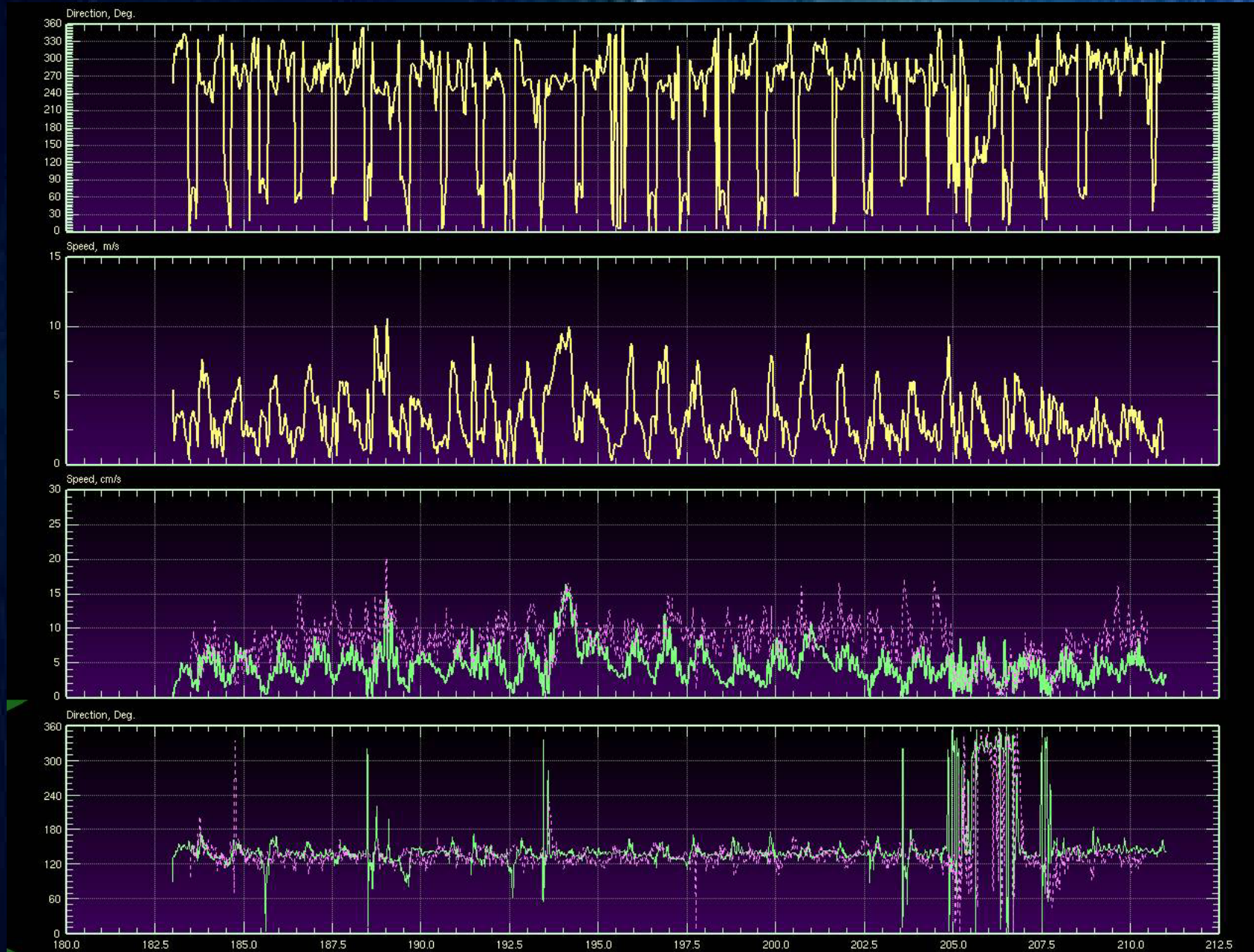
Scatter-Plot of Model vs. ADCP, Deep Station



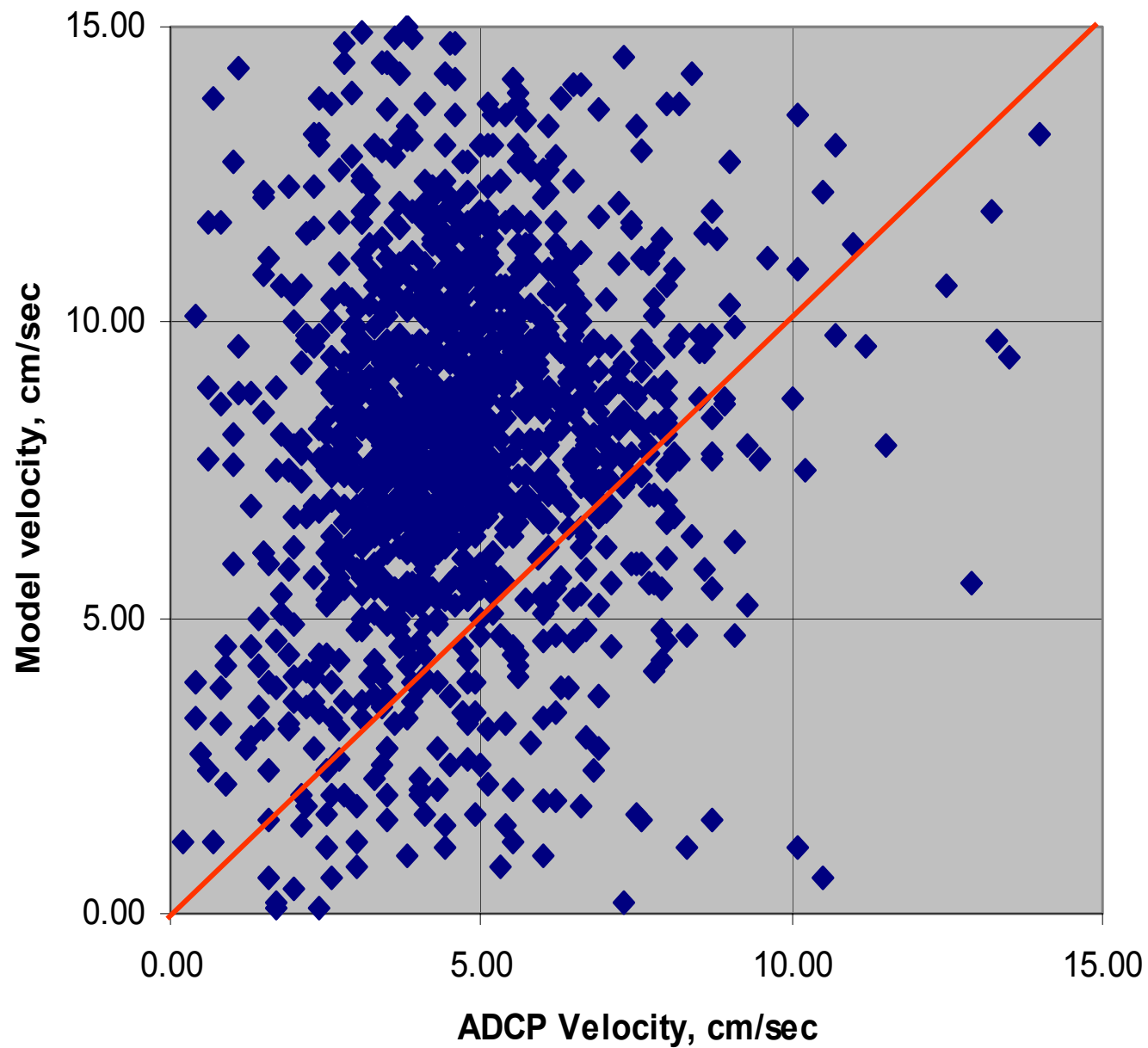
Wind Speed vs Velocities, Deep Station



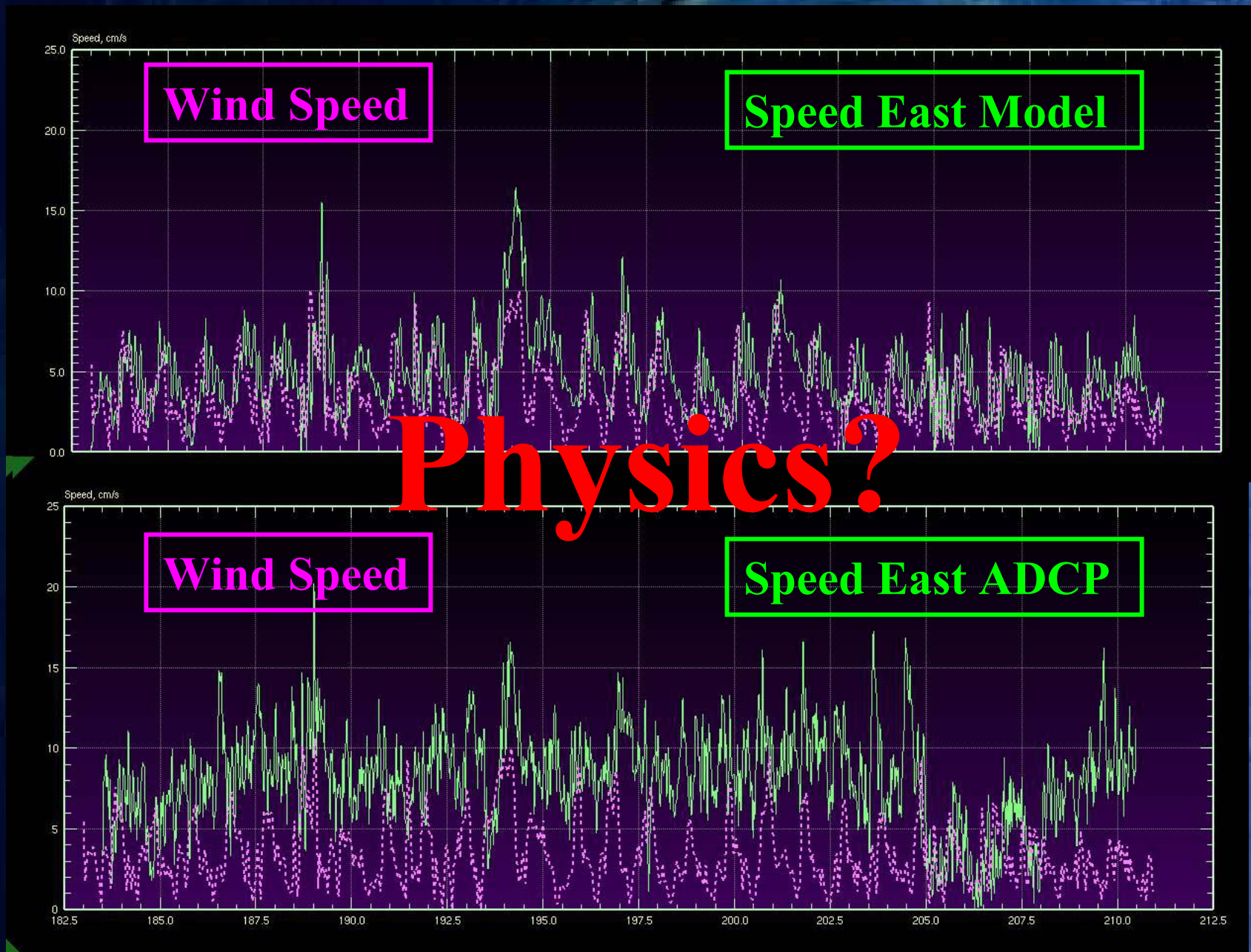
Model Results vs. ADCP Observations at Shallow (East) Station



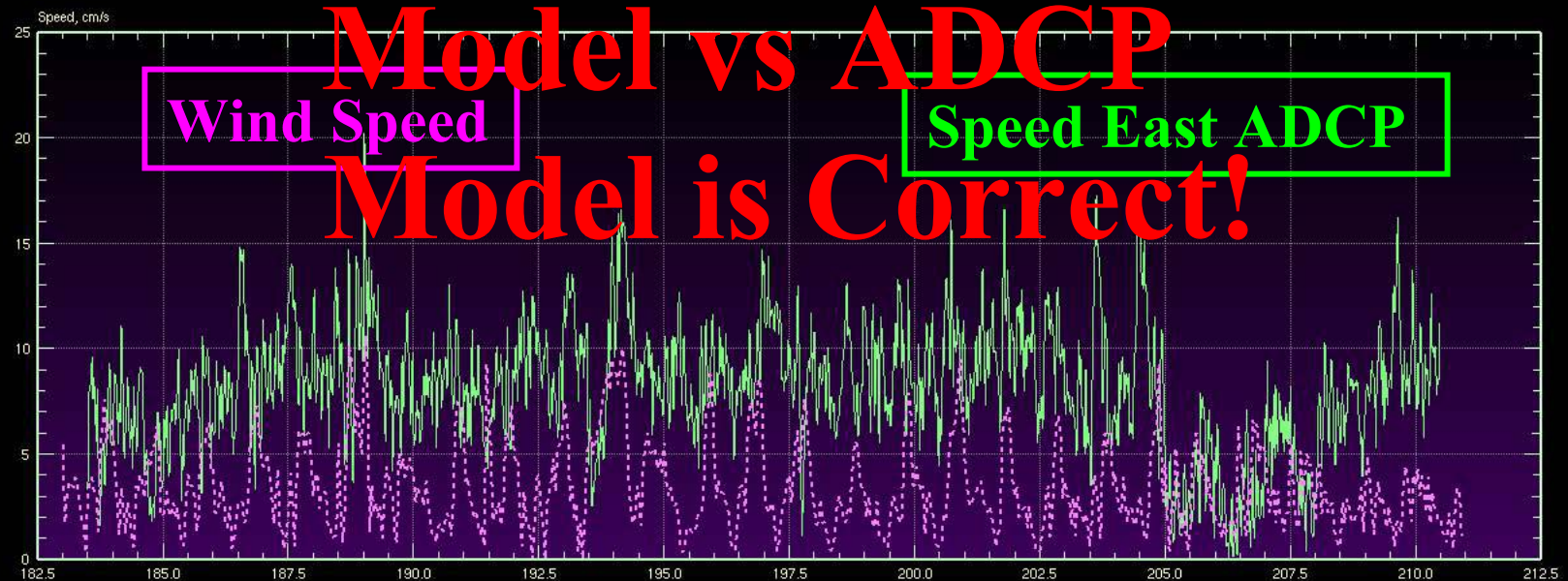
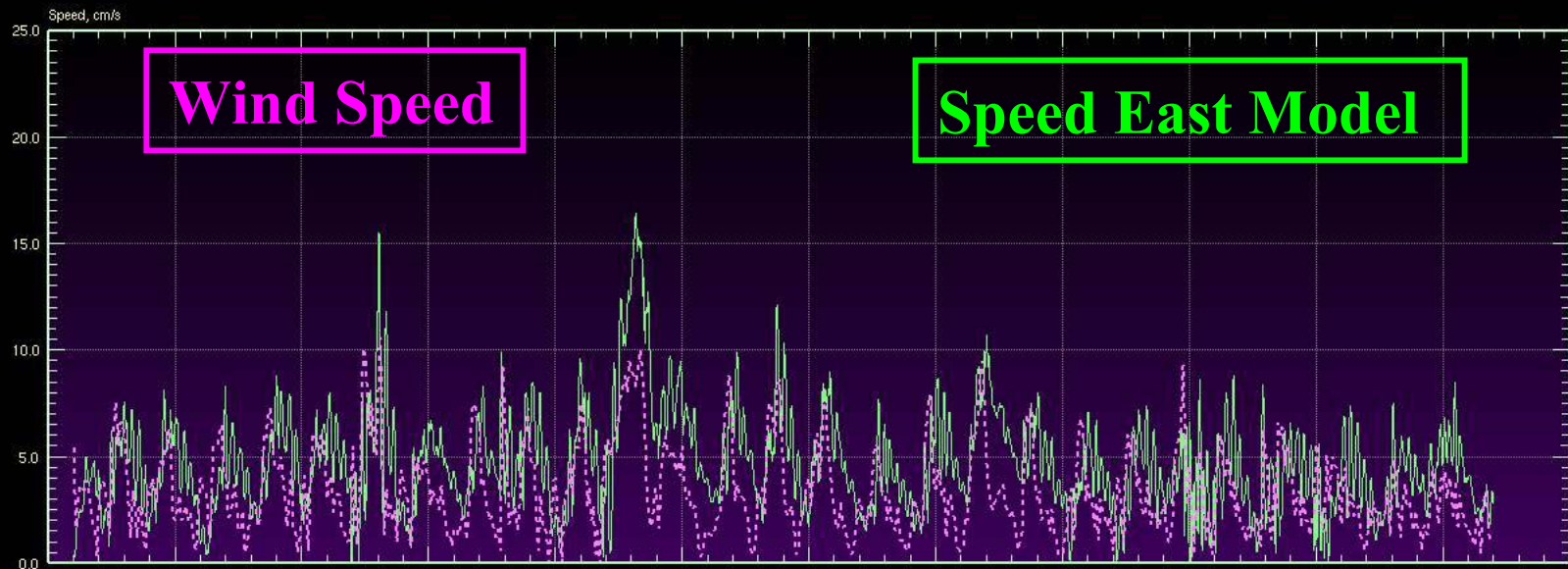
Scatter-Plot Model velocity vs. ADCP, Shallow



Correlations with wind speed



Correlations with wind speed



Model vs ADCP
Model is Correct!

Take Home Message:

**Field Data Do not Necessarily
Represent the Truth.**

**Field Data Must be Consistent with
the Correct Physics!**

**There might be hidden messages in
the data!**

Conclusion

- The UnTRIM numerical model is used to reproduce the wind circulation in Upper Klamath Lake (UKL).
- Circulation in Upper Klamath Lake is shown to be completely controlled by wind.
- The ADCP data at a deep station is reproduced reasonably well; at the shallow station, data are shown to be suspect.
- Discrepancies are due to the uncertainty in wind records and the assumption of uniform wind used to drive the model

Recommendation

- **Design a network of wind measurements over the entire lake region**
- **Measure wind carefully, do not treat it as ancillary data**
- **Re-evaluate Wind-Driven Circulation in Upper Klamath Lake by model.**
- **Add water quality components to model.**
 - Quantify Thermal Cycles**
 - Quantify Oxygen Cycles**
 - Identify causes of fish kills**

謝謝！



Thank You!